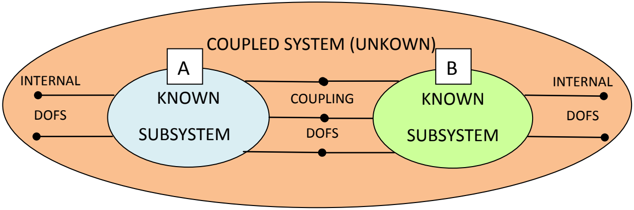
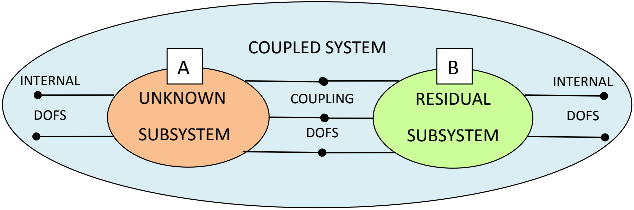
Experimental dynamic substructuring

- Information about at least one subsystem comes from tests
  - Coupling problems (addition / assembly of subsystems)

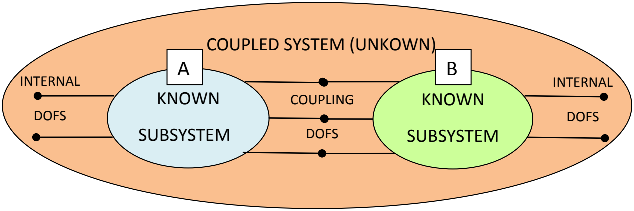


  - Decoupling problems (subtraction / disassembly of systems)



Coupling

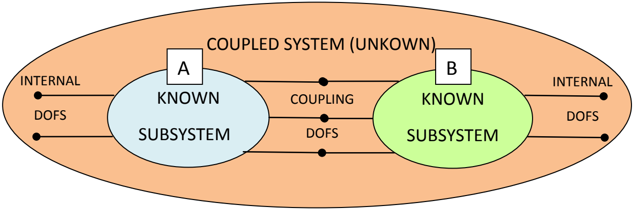
Methods for substructure coupling in the frequency domain



- Direct impedance coupling
- FRF-based substructuring (Jetmundsen, Bielawa & Flannelly)
- Domain decomposition (de Klerk, Rixen & Voormeren)
  - primal formulation in the frequency domain
  - dual formulation in the frequency domain

Coupling

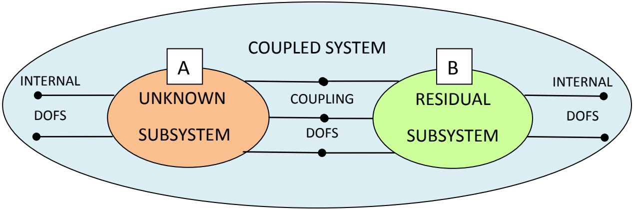
Typical issues in coupling problems



- Necessary to consider **all** interface DoFs
  - problems with rotational Dofs
- Modal truncation
- Inconsistency between FRFs
- Subsystems with very high / low stiffness / mass ratios

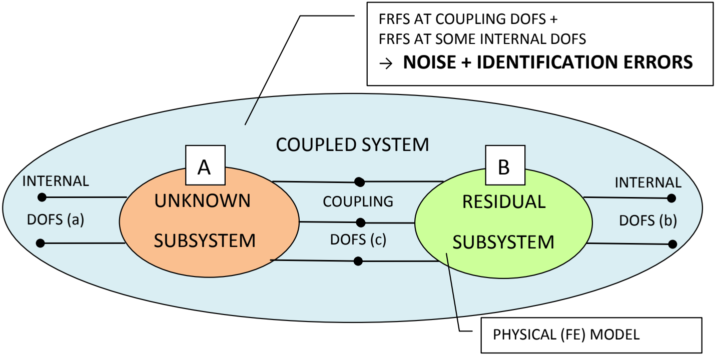
The same issues (and many more) are shared by decoupling problems

Decoupling: why is it important?



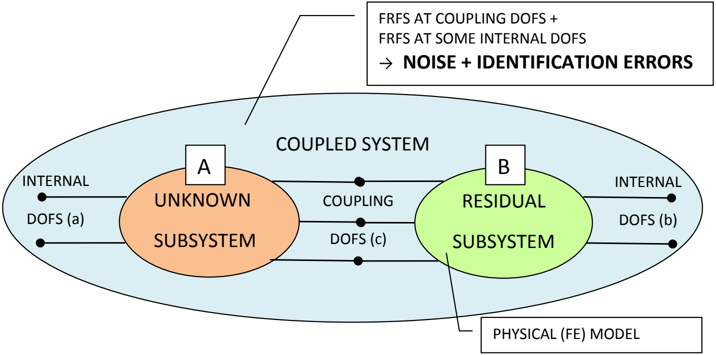
- Identify the dynamic behavior of a subsystem belonging to or embedded in a larger system
- Example of common applications
  - mass cancellation (it works very well)
  - joint identification (more difficult)
- Devised applications
  - identify parts of systems in operational conditions
  - diagnostics (monitoring of critical parts that can not be accessed easily)

Statement of decoupling problem



- Find the FRF of unknown substructure *A* starting from the FRF of the coupled system *AB*
- Subsystem *A* can be extracted from the coupled system *AB* by taking out the dynamic stiffness of the residual subsystem *B*

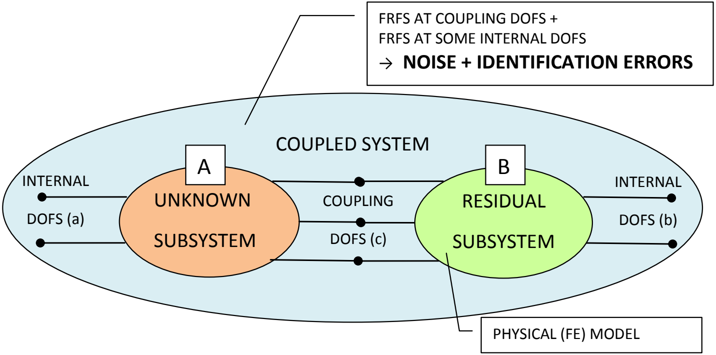
Statement of decoupling problem



Decoupling / subtraction / disassembly of structures seen as

1. inverse coupling, i.e. the way back after coupling / addition / assembly
   - gives rise to impedance and mobility approaches
   - equations for the coupling problem are rearranged to isolate (as unknown) one of the substructures instead of the coupled structure

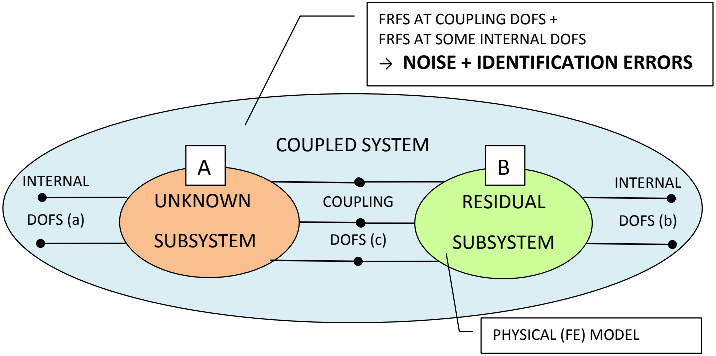
Statement of decoupling problem



Decoupling / subtraction / disassembly of structures seen as

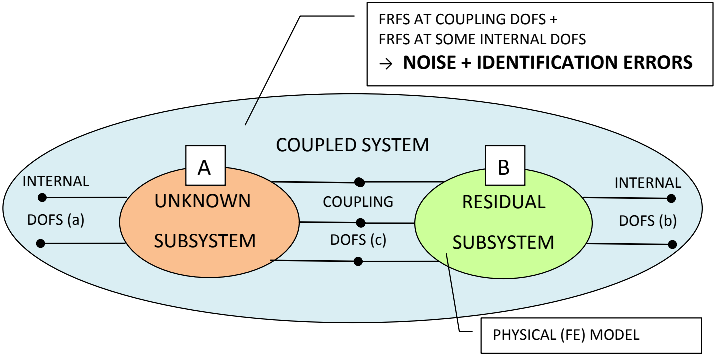
2. direct decoupling / subtraction / disassembly
   - gives rise to dual decoupling
   - accomplished by adding to the coupled system *AB* a fictitious subsystem with a dynamic stiffness opposite to that of the residual subsystem *B*

Statement of decoupling problem



- DoFs of the coupled system partitioned into
  - internal DoFs (not belonging to the couplings) of subsystem A (a)
  - internal DoFs of subsystem B (b)
  - coupling DoFs (c)

Statement of decoupling problem



- Which are the interface DoFs between the coupled system *AB* and the fictitious subsystem?
  - *standard interface*, including only the coupling DoFs (*c*) between subsystems *A* and *B* (quite natural for inverse coupling approaches)
  - *extended interface*, including also some internal DoFs ($i \subseteq b$) of the residual substructure *B*

Impedance approach

D'Ambrogio & Fregolent [Eurodyn 2005 (no internal DoFs), IMAC 2008]

$$\left[H^A\right]_{cc} = \begin{bmatrix} [I]_{cc} - \left[H^{AB}\right]_{cc}\left[\hat{Z}^B\right]_{cc} - \left[H^{AB}\right]_{ci}\left[\hat{Z}^B\right]_{ic} \\ -\left[H^{AB}\right]_{ic}\left[\hat{Z}^B\right]_{cc} - \left[H^{AB}\right]_{ii}\left[\hat{Z}^B\right]_{ic} \end{bmatrix}^{+} \begin{bmatrix} \left[H^{AB}\right]_{cc} \\ \left[H^{AB}\right]_{ic} \end{bmatrix}$$

- condensed dynamic stiffness $[\hat{Z}^B]$ of the residual subsystem B required
- $[\hat{Z}^B]$ meaningless at the resonant frequencies of the residual subsystem B with DoFs c and i grounded (see afterwords)

⇒ ill-conditioning around such frequencies

- If only coupling DoFs c are "measured", it is:

$$\left[H^A\right]_{cc} = \left([I]_{cc} - \left[H^{AB}\right]_{cc}\left[\hat{Z}^B\right]_{cc}\right)^{-1}\left[H^{AB}\right]_{cc} \qquad (*)$$

Mobility approach

D'Ambrogio & Fregolent [IMAC 2008 (introduction)]
Sjövall & Abrahamsson [MSSP 2008 (introduction and general antiresonances)]
Voormeren & Rixen [IMAC 2009 (interpretation)]

$$\left[H^A\right]_{cc} = \left[\left[H^{AB}\right]_{cc} \quad \left[H^{AB}\right]_{ci}\right] \left(\left[[I]_{cc} \quad [0]_{ci}\right] - \begin{bmatrix} \left[H^B\right]_{cc} \\ \left[H^B\right]_{ic} \end{bmatrix}^{+} \begin{bmatrix} \left[H^{AB}\right]_{cc} & \left[H^{AB}\right]_{ci} - \left[H^B\right]_{ci} \\ \left[H^{AB}\right]_{ic} & \left[H^{AB}\right]_{ii} - \left[H^B\right]_{ii} \end{bmatrix} \right)^{+}$$

- No visible source of ill-conditioning, but ill-conditioning observed around the same frequencies as for impedance
- Explained by considering *general antiresonances*, i.e. frequencies at which

$$\det \left(\begin{bmatrix} \left[H^B\right]_{cc} & \left[H^B\right]_{ci} \\ \left[H^B\right]_{ic} & \left[H^B\right]_{ii} \end{bmatrix} \right) = 0$$

- If only coupling DoFs c are "measured"

$$\left[H^A\right]_{cc} = \left[H^{AB}\right]_{cc} \left([I]_{cc} - \left(\left[H^B\right]_{cc}\right)^{-1} \left[H^{AB}\right]_{cc} \right)^{-1}$$

$$\left(\left[H^B\right]_{cc}\right)^{-1} = \left[\hat{Z}^B\right]_{cc} \quad \Rightarrow \quad \text{(r.h.s above)} = (^*)^T$$

Dual formulation

de Klerk, Rixen & de Jong [IMAC 2006 (standard interface)
D'Ambrogio & Fregolent [IMAC 2009 (extended interface)]

In dual formulation, the union between the coupled system *AB* and the fictitious subsystem can be written as:

$$\begin{bmatrix} [Z^{AB}] & [0] & [B^{AB}]^T \\ [0] & -[Z^{B}] & [B^{B}]^T \\ [B^{AB}] & [B^{B}] & [0] \end{bmatrix} \begin{Bmatrix} \{u^{AB}\} \\ \{u^{B}\} \\ \{\lambda\} \end{Bmatrix} = \begin{Bmatrix} \{f^{AB}\} \\ \{f^{B}\} \\ \{0\} \end{Bmatrix}$$

where:

$[Z^{(AB)}]$, $-[Z^{(B)}]$:	dynamic stiffness matrices of coupled and fictitious system
$\{u^{(AB)}\}$, $\{u^{(B)}\}$:	vectors of degrees of freedom
$\{f^{(AB)}\}$, $\{f^{(B)}\}$:	external force vectors
$[B^{AB}]$, $[B^{B}]$:	Boolean constraint matrices extracting the (standard or extended) interface DoFs among the full set of DoFs
$\{\lambda\}$:	Lagrange multipliers corresponding to connecting force intensities