

## ***Modal response of composite laminate plates.***

In this project the prediction of resonant frequencies for the modal response of eight-ply laminated composite plates is validated. This work is performed in support of the Los Alamos Damage Prognosis project, see Reference [1].

Figure 1 shows the composite plates. Each plate is 152.0 mm (6.0 inch) square, 1.0 mm (0.04 inch) thick, and made of eight orthotropic carbon fiber plies. Each ply is 0.127 mm (0.005 inch) thick. The ply orientation angles are [0; 45; 90; -45; -45; 90; 45; 0] degrees. Modal tests are conducted on a population of eight nominally identical plates, as shown in Figure 2. The test-to-test variability and unit-to-unit variability are fully characterized using sample statistics and principal component analysis. The first five modes are identified with mean frequency values of 107.4 Hertz, 191.8 Hertz, 274.1 Hertz, 315.3 Hertz, and 398.9 Hertz.



*Figure 1. Population of eight nominally identical laminated composite plates.*

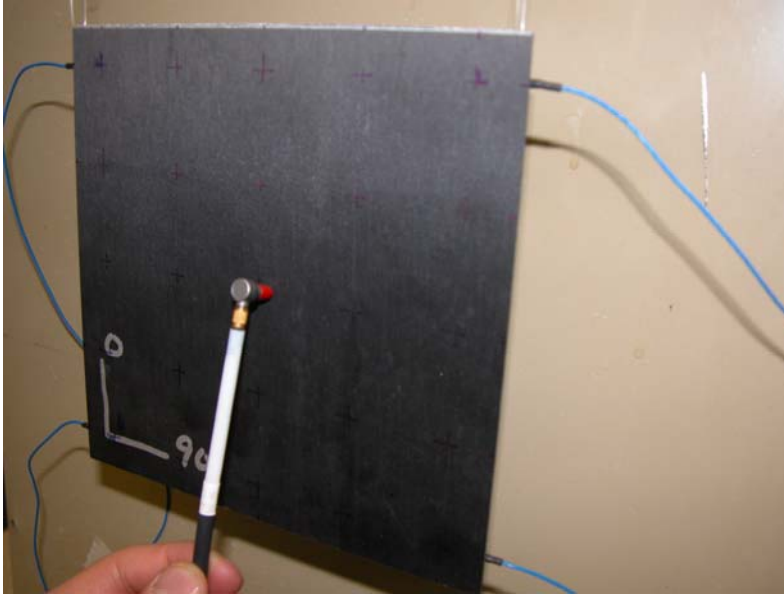


Figure 2. Laminated composite plate being impacted for modal testing.

A Finite Element Model (FEM) is implemented and analyzed using the general-purpose finite element package HKS/ABAQUS™ [2]. The plate is meshed with 20-node quadratic hexahedra, with ten elements in each in-plane direction and one element through the thickness of each ply. The FEM includes a multi-length scales constitutive material model capable of predicting composite damage modes such as delamination and fiber splitting [3]. Figure 3 shows a mode shape obtained during the code verification exercise. Using the nominal geometry and material properties, natural frequencies are predicted at 107.5 Hertz, 205.7 Hertz, 278.1 Hertz, 334.0 Hertz, and 411.8 Hertz.

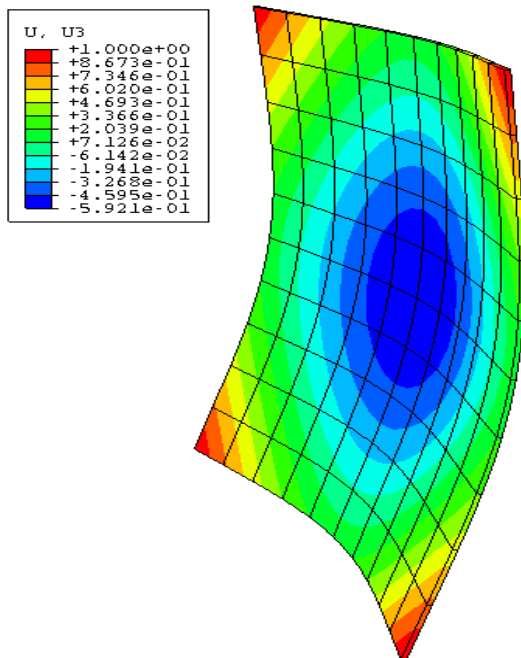


Figure 3. Mode shape predicted by the finite element model.

The V&V work includes verifying some implementation aspects of the code; mesh convergence studies; effect screening and analyses of variance based on designs of computer experiments;

polynomial meta-modeling; and the sampling-based propagation of uncertainty. Each step verifies or validates a specific aspect of the numerical simulation. Each step also quantifies one of the independent sources of uncertainty that contribute to the overall discrepancy between measurements and predictions. The technology and some of the tools applied to V&V are detailed in References [4, 5]. The sources of uncertainty originating from coding mistakes, calculation verification, model fitting, and parametric variability are summarized in Table 1. Test measurements are compared to predictions, and statistics of experimental variability and test-analysis correlation are also listed in Table 1. Details of these assessments are provided in Reference [6].

Table 1. Summary of the quantification of testing and modeling uncertainty.

Modeling Uncertainty	Mean ( $\mu$ )	Standard Deviation ( $\sigma$ )
Code Error, Solution Convergence	$\mu_1 = 1.49$ Hertz	$\sigma_1 = 0.87$ Hertz
Meta-model Fitting Error	$\mu_2 = 2.15$ Hertz	$\sigma_2 = 0.18$ Hertz
Model Parameter Variability	$\mu_3 = 0.00$ Hertz	$\sigma_3 = 19.00$ Hertz
Total Prediction Error	$\mu_T = 3.64$ Hertz	$\sigma_T = 19.02$ Hertz
Experimental Uncertainty	Mean ( $\mu$ )	Standard Deviation ( $\sigma$ )
Experimental Variability	$\mu_E = 0.00$ Hertz	$\sigma_E = 2.49$ Hertz
Test-analysis Correlation	$\mu_C = 8.97$ Hertz	$\sigma_C = 19.18$ Hertz

Statistics are averaged over the five modal frequencies for simplicity. A naïve aggregation of the independent sources of modeling uncertainty leads to a total prediction error (mean and standard deviation) of:

$$\mu_T = \mu_1 + \mu_2 + \mu_3 = 3.64 \text{ Hertz}, \quad \sigma_T^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = (19.02 \text{ Hertz})^2$$

The total prediction error model ( $\mu_T; \sigma_T$ ) builds the uncertainty from the bottom up by aggregating various independent sources of modeling uncertainty. On the other hand the test-analysis correlation error model ( $\mu_C; \sigma_C$ ) is a top-level assessment. Both are needed to assess the prediction accuracy of the composite model. The first observation is that the bias error ( $\mu_C - \mu_T$ ) = 5.33 Hertz is due to the residual modeling error, that is, the error made when the composite model is substituted to reality. The second observation is that variability could be significantly reduced by better characterizing the composite lay-up and material properties. Reference [6] also shows that calibration can be taken advantage of to infer the values of input parameters that cannot be measured directly through coupon testing.

This study is the first phase of a hierarchy of three validation experiments defined to assess various aspects of the modeling of multi-layered composite plates. Modal testing is used to extract low-frequency resonant modes, from which the linear response can be validated. Quasi-static loading will be used to validate the large displacement response, although without fully exercising the fiber splitting and delamination damage modes nor the Cohesive Zone Model algorithm. Impact testing will be used to validate the modeling of the high-frequency, transient wave propagation coupled with all aspects (constitutive and damage models, numerical algorithms) of the composite damage models.

[1] Farrar, C.R., Wait, J.R., Tippetts, T.B., Hemez, F.M., Park, G., Sohn, H., "Damage Detection and Prediction for Composite Plates," *2004 Conference on Material Science and Technology (MST)*, Symposium on Materials Damage Prognosis, New Orleans, Louisiana, September 26-29, 2004. LA-UR-04-8386.

[2] Abaqus™/Explicit, User's Manual, Version 6.2, Hibbitt, Karlsson & Sorensen, Pawtucket, Rhode Island, 2001.

- [3] Tippetts, T.B., Hemez,, F.M., "Non-Linear Models of Composite Laminates," *23<sup>rd</sup> SEM International Modal Analysis Conference (IMAC-XXIII)*, Orlando, Florida, January 31-February 3, 2005. LA-UR-04-8100.
- [4] Hemez, F.M., Doebling, S.W., Anderson, M.C., "A Brief Tutorial on Verification and Validation," *22<sup>nd</sup> SEM International Modal Analysis Conference*, Dearborn, Michigan, January 26-29, 2004. LA-UR-03-8491.
- [5] Kerschen, G., Golinval, J.C., Hemez, F.M., "Bayesian Model Screening for the Identification of Non-linear Mechanical Structures," *ASME Journal of Vibration and Acoustics*, Vol. 125, July 2003, pp. 389-397. LA-UR-02-2787.
- [6] Hemez, F.M., Tippetts, T.B., "Verification and Validation of a Composite Model," *Internal Report*, Los Alamos National Laboratory, Los Alamos, New Mexico, December 2004. LA-UR-04-8195.