Modal Topics: Sensor Performance, A (Hopefully) Short Primer*

Michael Todd
Dept. of Structural Engineering
University of California, San Diego
mdt@ucsd.edu

Email me if you want a copy.

**We only have 90 minutes, so I think we're pretty safe.*

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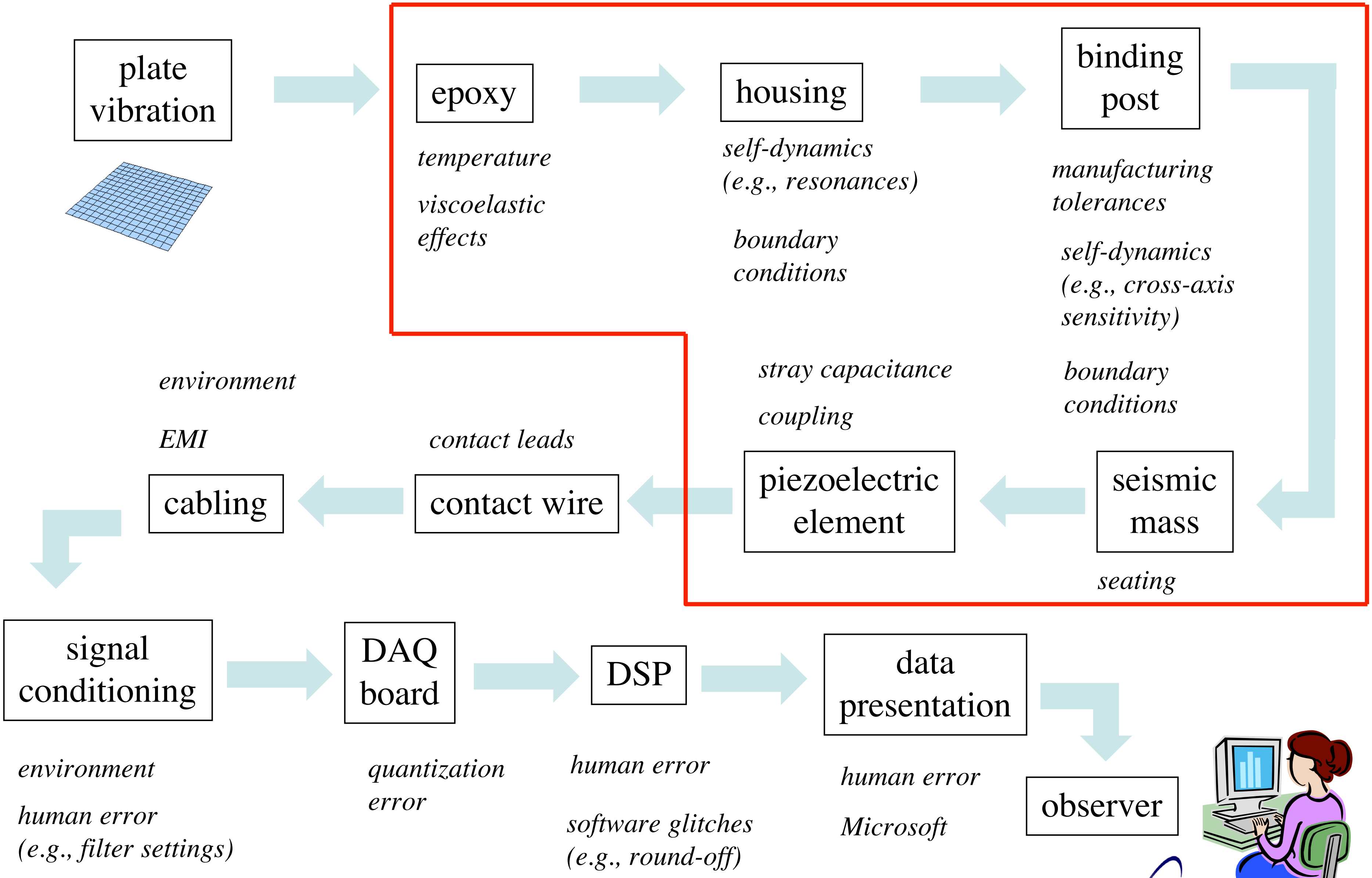
Outline

- Sensors as dynamic systems
- Zero-order sensors
- First-order sensors
- Second-order sensors
- Sensor performance metrics
  - sensitivity
  - bandwidth/transmission band(s)
  - cross-axis sensitivity
  - sensitivity to extraneous measurands
- Fiber Bragg Grating sensing
  - operational principles
  - deployment examples

The Process of Getting An Acceleration Measurement



Input/Output Representation of Sensors

What we WANT to measure is some kind of response of the plate (e.g., acceleration at point q, $Y_q(\omega)$), to some kind of excitation of the plate (e.g., impact hammer strike at point p, $X_p(\omega)$); we usually describe the response in terms of an FRF (frequency response function):

$$H_{pq}(\omega) = Y_q(\omega) / X_p(\omega)$$

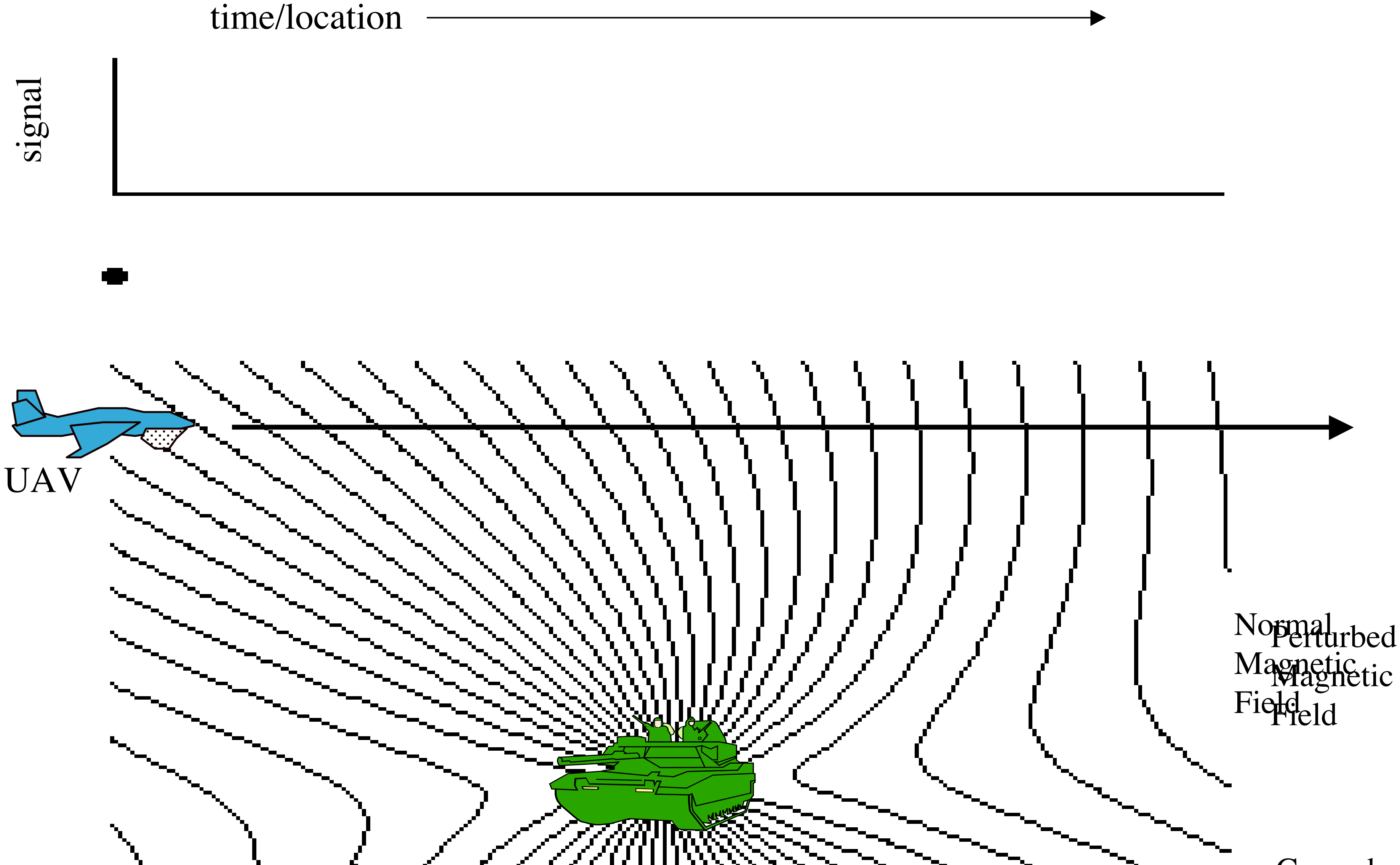
Just as the plate *transfers* an excitation into an acceleration, the accelerometer subsequently *transfers* this acceleration into a measured representation of itself. Thus, what we ACTUALLY measure ($Y_{q,obs}(\omega)$) is NOT necessarily equal to what we WANT to measure ($Y_q(\omega)$):

$$Y_{q,obs}(\omega) = \underbrace{\left(H_{epoxy}(\omega)H_{housing}(\omega)H_{post}(\omega)H_{mass}(\omega)H_{piezo}(\omega)H_{wires}(\omega)\right)}_{H_{sensor}(\omega)}Y_q(\omega)$$

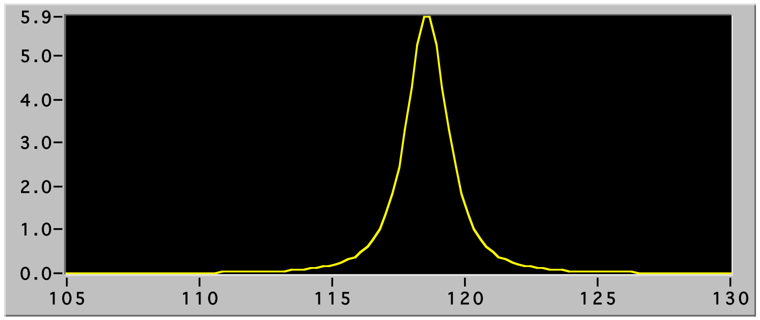
And this doesn't include all data acquisition and signal conditioning response functions!

We need to understand the behavior of the sensor transfer function in order to choose the right sensor characteristics for an application and calibrate it.

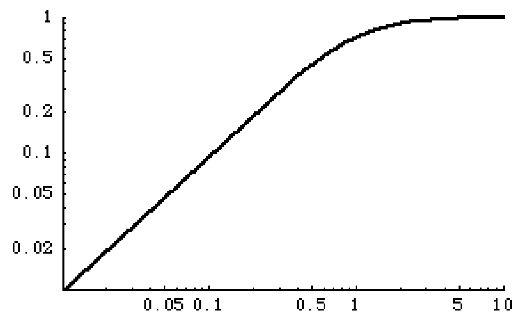
An Example to Consider...Detecting a Tank in the Trees

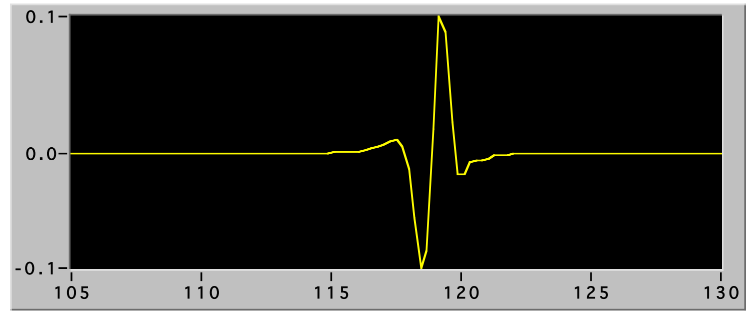


Tank Example (Continued)

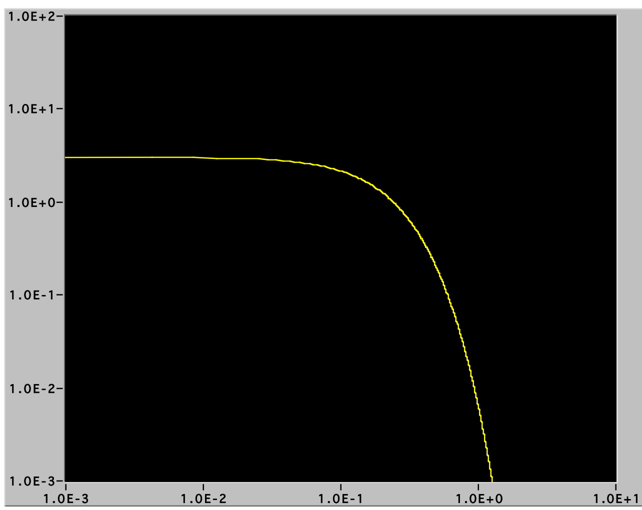


magnetic field time history

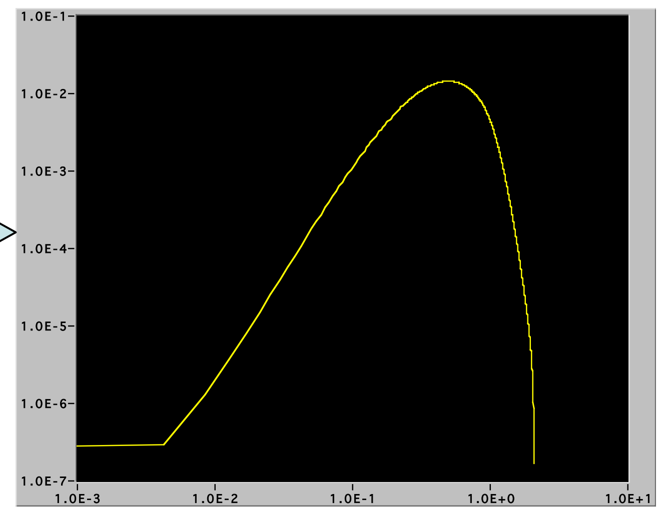




what the sensor gives you



sensor transfer function

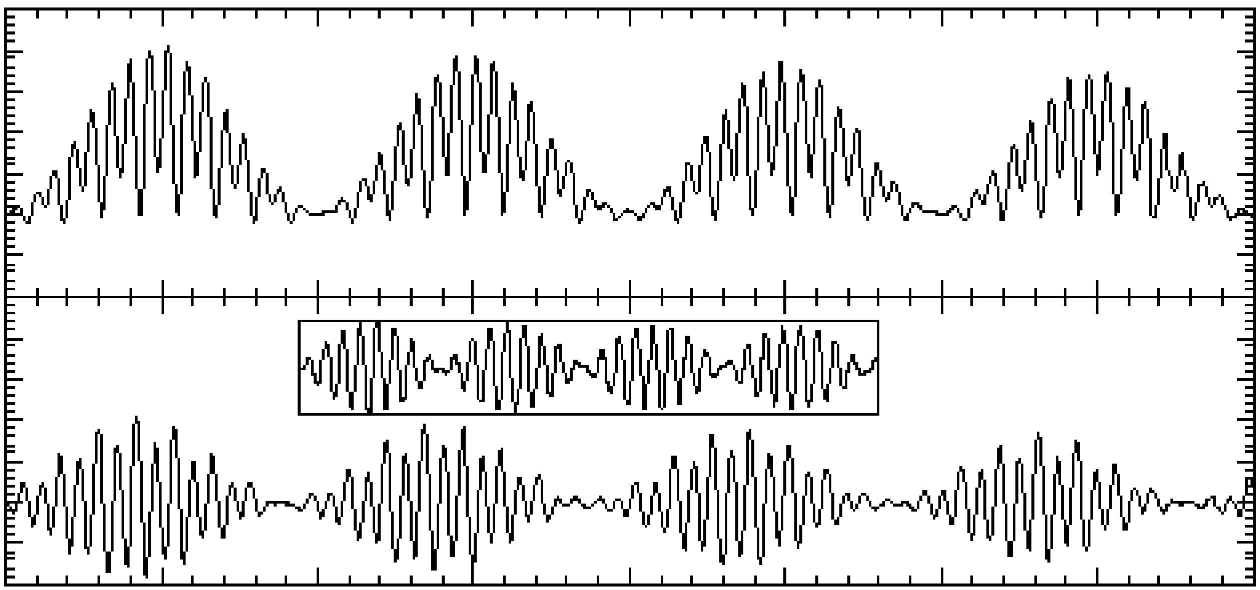


Another "Real" Example...Measuring Pendulation Loads

Sea state can lead to dangerous load pendulation during LOTS exercises

Design fiber optic accelerometer array to monitor load and ship motions

Also asked to monitor centripetal acceleration (has DC component!)

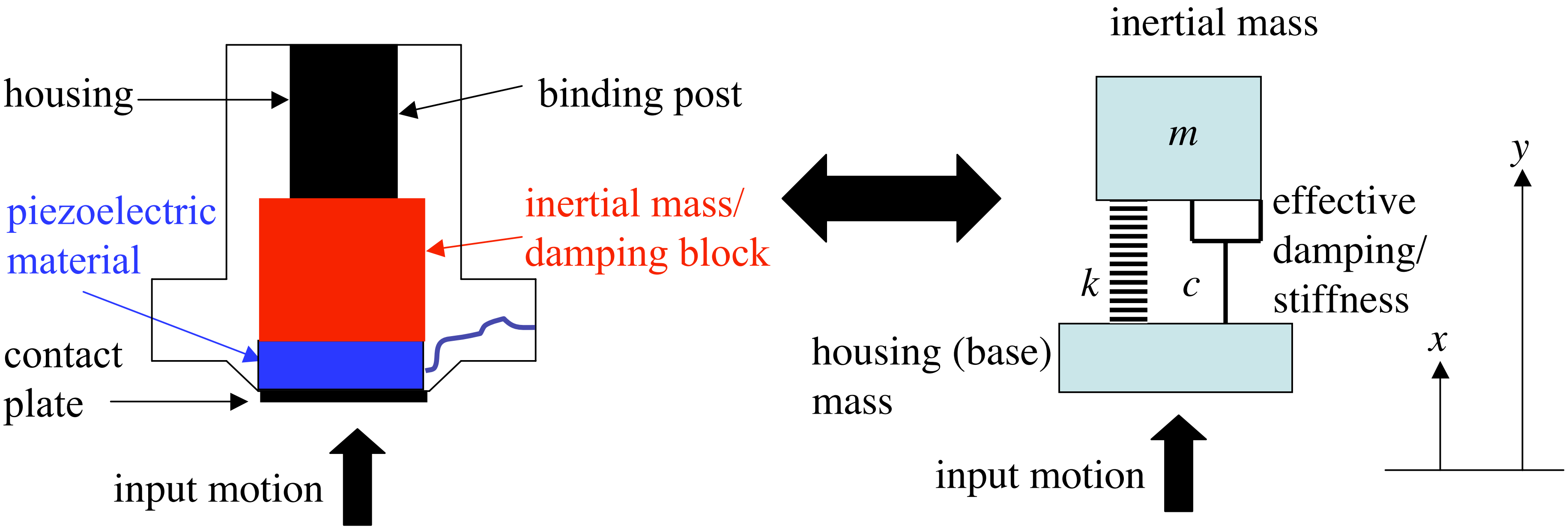


Actual signal

What we measured

electronics had low-frequency attenuation!

General Input/Output Sensor Model: Accelerometer



Newton's 2nd Law on m:

$$\underbrace{m\frac{d^2y}{dt^2}+c\frac{dy}{dt}+ky}_{\text{"output"}}=\underbrace{c\frac{dx}{dt}+kx}_{\text{"input"}}$$

And, in general,

$$a_n\frac{d^ny}{dt^n}+a_{n-1}\frac{d^{n-1}y}{dt^{n-1}}+\ldots+a_1\frac{dy}{dt}+a_0y=F(t)$$

Sensors are themselves dynamical systems!

Zero-Order Sensors

These sensors are NOT dynamic systems and can only measure static signals

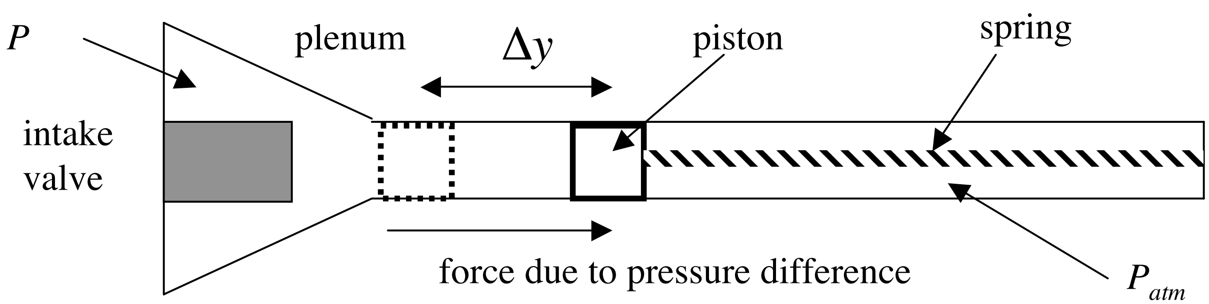
sensor model is $a_0 y(t) = F(t)$ so the output is easily obtained $y(t) = \underbrace{(1/a_0)}_{K \equiv \text{ static sensitivity}} F(t)$

- The sensor's static sensitivity K determines the simple proportional input/output relationship.

- The sensor has no dynamics, so it responds instantly to the change in $F(t)$

- Sensor ideally contains no storage or inertial elements (or, more accurately, the behavior of these elements is negligible)

- If $F(t)$ is a dynamic signal, then the reading of the sensor (y) can ONLY be considered valid when the sensor is at static equilibrium



$$P - P_{atm} = -kA_{piston}\Delta y, \quad \text{or}$$

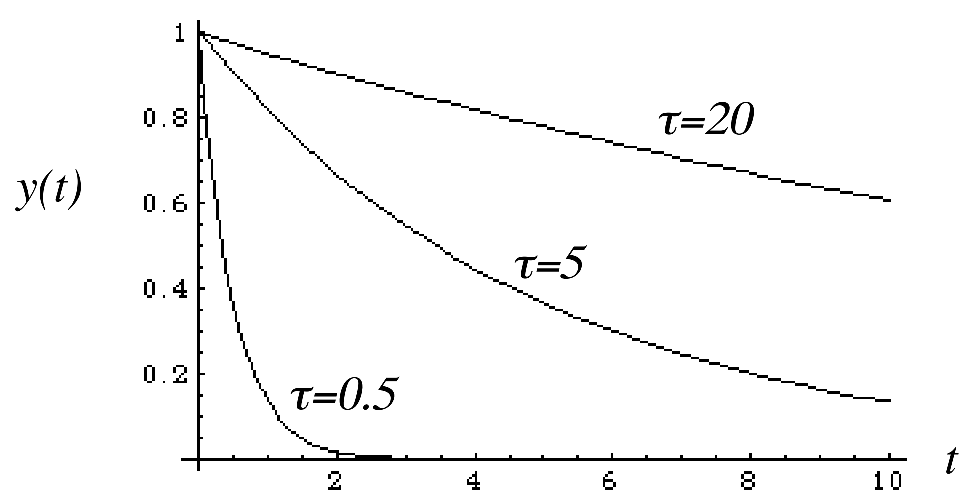
$$\Delta P = \underbrace{-kA_{piston}}_{\text{static sensitivity, } K} \Delta y$$

First-Order Sensors

- These sensors contain storage elements which cannot respond immediately to inputs
- Modeled as first-order dynamical system $a_1 \frac{dy}{dt} + a_0 y = F(t)$

$$\underset{\text{time constant}}{\tau} \frac{dy}{dt} + y = \underset{\text{static sensitivity}}{K} F(t)$$

- The zero-state response (with no input) of this system shows the influence of τ:



The larger the sensor's time constant, the longer it takes to respond to input changes.

First-Order Sensors (Cont'd)

• We know from dynamic systems theory that we can utilize the powerful kernel method based on zero-state impulse-response to get the formula for the response to ANY input. The impulse response for an impulse-loaded first-order system

$$\tau \frac{dy}{dt} + y = K\delta(t)$$

is $$h_1(t) = Ke^{-t/\tau}/\tau$$

For any general input *F(t)*, then, we know the sensor response to it:

$$y(t) = y_0 e^{-t/\tau} + \int_{T=0}^{t} F(T) h_1(t-T)\, dT$$

$$= \underbrace{y_0 e^{-t/\tau}}_{\substack{\text{transient}\\ \text{response}}} + \underbrace{\int_{T=0}^{t} F(T)(K/\tau) e^{-(t-T)/\tau}\, dT}_{\substack{\text{steady-state}\\ \text{response}}}$$

Example: Bulb Thermometer

The change in energy stored in the bulb, $\dot{E}$, must balance the exchange of heat between sensor and environment:

$$\dot{E} = \dot{Q}$$

$$mc\frac{dT}{dt} = hA(T_\infty - T)$$

$$\underbrace{\frac{mc}{hA}}_{\tau}\frac{dT}{dt} + T = T_\infty$$

$$\tau\frac{dT}{dt} + T = F(t)$$

m = liquid mass in the sensor
c = specific heat of liquid
h = convective heat transfer coefficient
A = sensor surface area

- Standard from for first-order sensor
- Note that the static sensitivity, K, is 1

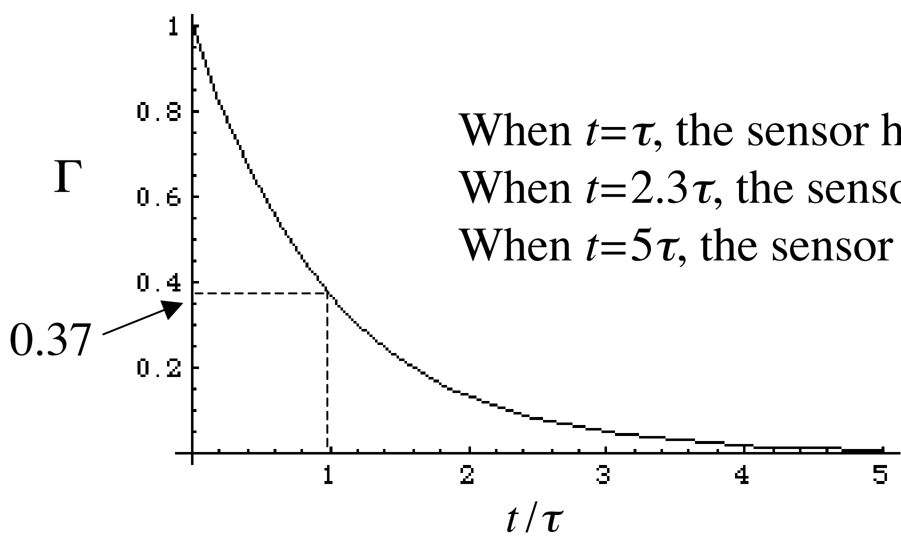
First-Order Sensors: Step Input

- Consider the sensor response to a step change in the input, say 0 to A, so $F(t)=A$:

$$y(t) = y_0 e^{-t/\tau} + \int_{T=0}^{t} A(K/\tau) e^{-(t-T)/\tau} dT = KA + (KA - y_0) e^{-t/\tau}$$

- Our steady response (what we want to measure) is KA, which is the actual measurement A modified by the constant static sensitivity of the sensor K
- Our current measurement (initial condition) is y_0
- We can define an error fraction function Γ as

$$\Gamma(t) = \frac{y(t) - y_{ss}}{y_0 - y_{ss}} = e^{-t/\tau}$$



When $t=\tau$, the sensor has responded to 63% of the real measurement;
When $t=2.3\tau$, the sensor has responded to 90% of the real measurement;
When $t=5\tau$, the sensor has responded to 99% of the real measurement

One may experimentally find τ by plotting the ln of error function during test data collection, since $\ln\Gamma = -(1/\tau)t$ and finding the slope of the best-fit line.

First-Order Sensors: Periodic Input

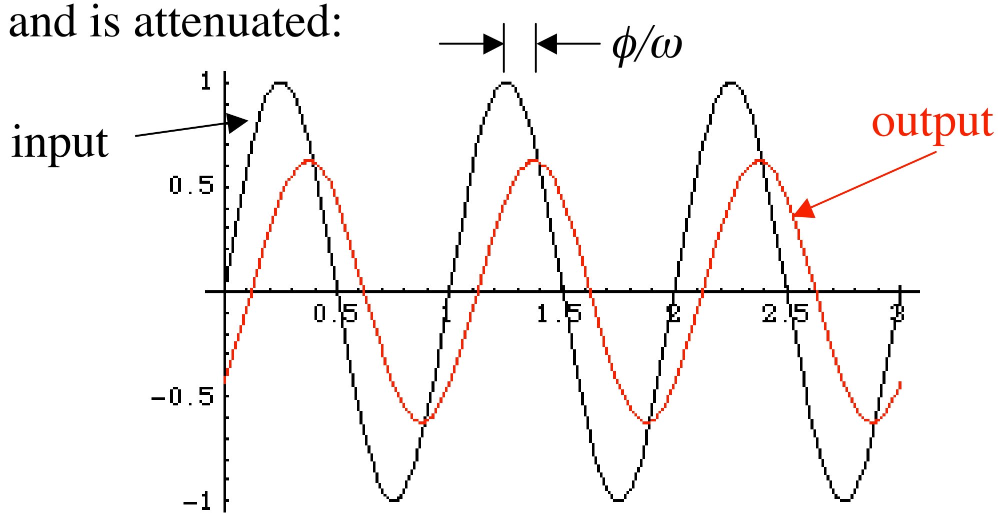
- Consider an input that is periodic in time, so $F(t)=A\sin\omega t$

$$y(t) = y_0 e^{-t/\tau} + \int_{T=0}^{t} A\sin\omega T\,(K/\tau)e^{-(t-T)/\tau}\,dT = Ce^{-t/\tau} + \frac{KA}{\sqrt{1+\tau^2\omega^2}}\sin(\omega t + \phi)$$

where $C = y_0 + \dfrac{KA\omega\tau}{1+\tau^2\omega^2}$, $\tan\phi = -\omega\tau$

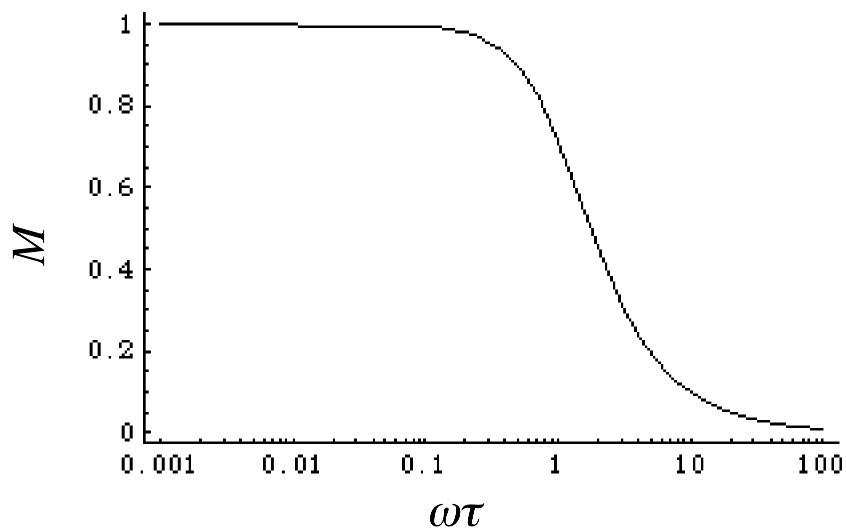
- Once the transient portion dies out, we have the first-order sensor frequency response to a periodic input

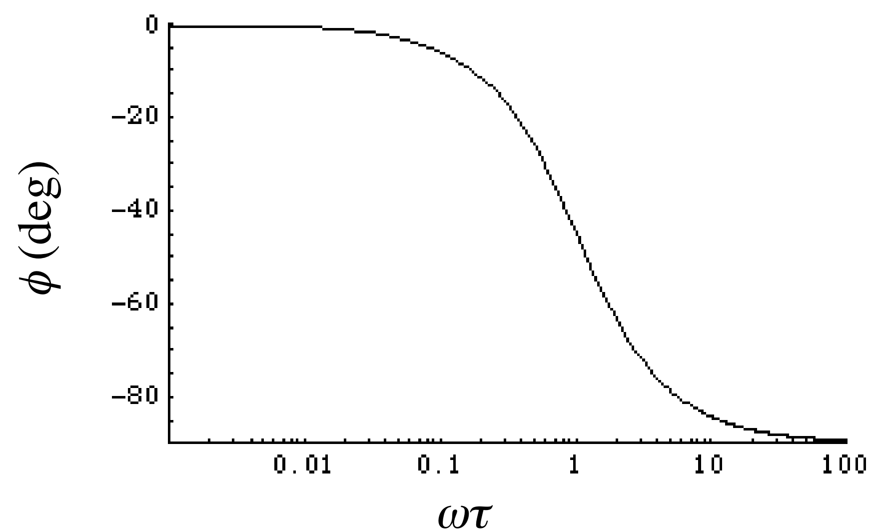
- The steady-state sensor response matches the input in frequency, but it lags the input by a time delay ϕ/ω and is attenuated:



First-Order Sensors: Periodic Input (Cont'd)

- This attenuation and lag, as a function of parameters ω and τ, are shown here





Attenuation factor M is the ratio of the dynamic amplitude to the static amplitude:

$$M = \frac{\dfrac{KA}{\sqrt{1+\tau^2\omega^2}}}{KA} = \frac{1}{\sqrt{1+\tau^2\omega^2}}$$

Phase shift was defined previously, $-\tan^{-1}\omega\tau$

- to measure high frequencies, need very small τ or too much attenuation (filtering)
- to measure low-frequency (towards static) signals, can use systems with large τ

Second-Order Sensors

- These sensors contain both storage elements (in the form of damping) as well as inertia

- Modeled as second-order dynamic systems:

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = F(t)$$

$$\frac{1}{\omega_n^2} \frac{d^2 y}{dt^2} + \frac{2\zeta}{\omega_n} \frac{dy}{dt} + y = KF(t)$$

$$\omega_n = \sqrt{a_0 / a_2} \equiv \text{ natural frequency}$$

$$\zeta = \mathrm{a}_1 / 2\sqrt{a_0 a_2} \equiv \text{damping ratio}$$

- The natural frequency corresponds to a time scale that the sensor "wants" to respond to inputs

- The damping ratio determines the characteristic manner in which the sensor response approaches the steady-state measurement:
  - $0 < \zeta < 1$ (underdamped) sensor transient responds will oscillate (at ω_n)
  - $\zeta > 1$ sensor transient response will monotonically (not oscillate) approach steady-state; $\zeta = 1$, or critical damping represents the quickest monotonic approach

Second-Order Sensors (Cont'd)

• The vast majority of sensor designs result in sub-critical damping (oscillations), so we will proceed with our discussion with that assumption. The transient response of a second-order sensor system with sub-critical damping is

$$y_{trans}(t) = e^{-\zeta\omega_n t}\left[y_0 \cos\omega_n\sqrt{1-\zeta^2}\,t + \left(\frac{\dot{y}_0 + y_0\omega_n\zeta}{\omega_n\sqrt{1-\zeta^2}}\right)\cos\omega_n\sqrt{1-\zeta^2}\,t\right]$$

sensor initial conditions

• We can again use the kernel method to get the general sensor response:

$$\frac{1}{\omega_n^2}\frac{d^2y}{dt^2} + \frac{2\zeta}{\omega_n}\frac{dy}{dt} + y = K\delta(t) \quad \text{results in} \quad h_2(t) = \frac{K\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t$$

• For any general input *F(t)*, then, we know the sensor response to it:

$$y(t) = y_{trans}(t) + \int_{T=0}^{t} F(T)h_2(t-T)\,dT$$

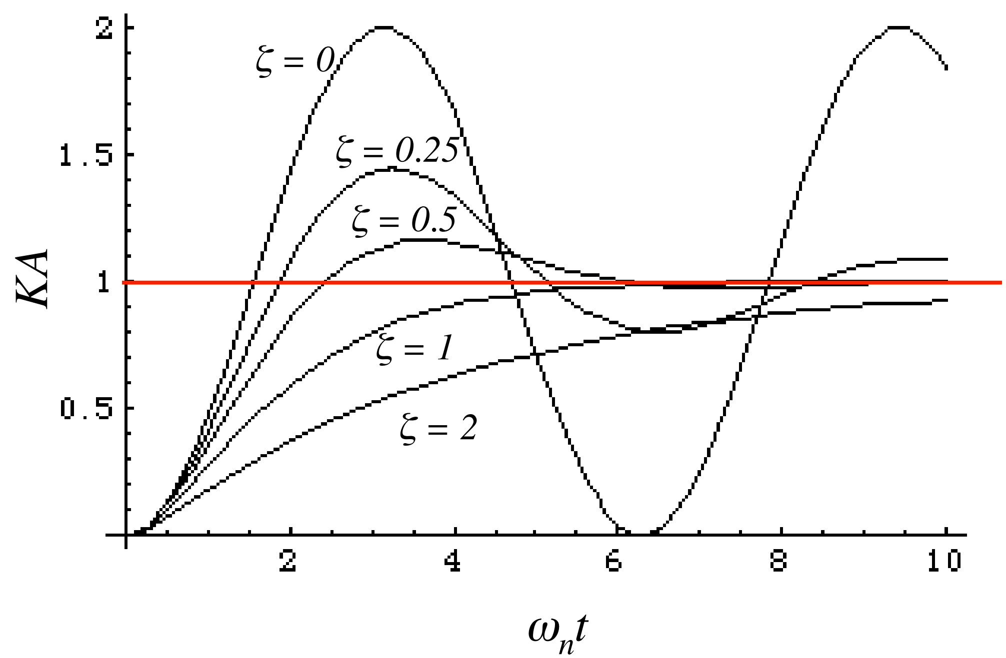
$$= \underbrace{y_{trans}(t)}_{\substack{transient \\ response}} + \underbrace{\int_{T=0}^{t} F(T)(K\omega_n/\sqrt{1-\zeta^2})e^{-\zeta\omega_n(t-T)}\sin\left[\omega_n\sqrt{1-\zeta^2}(t-T)\right]dT}_{\substack{steady-state \\ response}}$$

Second-Order Sensors: Step Input

- Consider the sensor response to a step change in the input, say 0 to *A*, so *F(t)=A:*

$$y(t) = y_{trans}(t) + \int_{T=0}^{t} A\left(K\omega_n / \sqrt{1-\zeta^2}\right) e^{-\zeta\omega_n (t-T)} \sin\left[\omega_n \sqrt{1-\zeta^2}(t-T)\right] dT$$

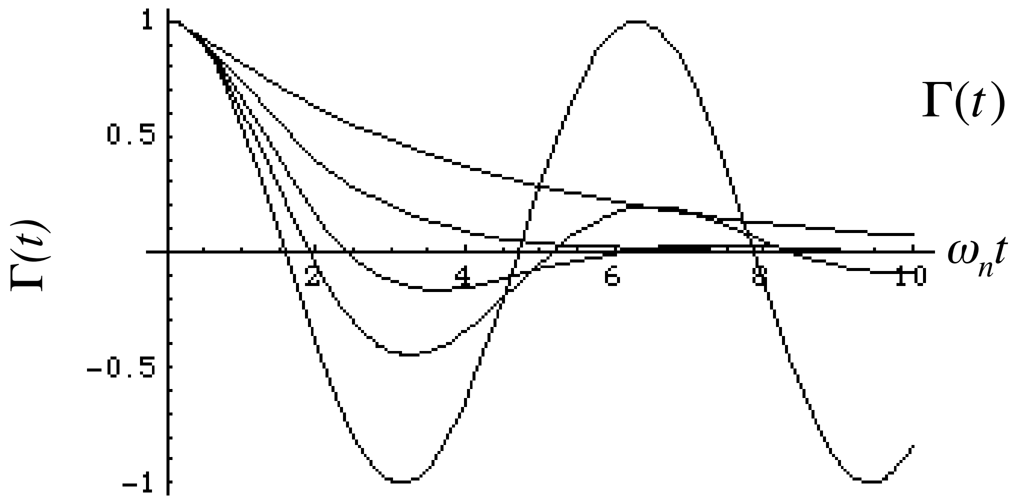
$$= KA - KAe^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right), \text{ where } \omega_d = \omega_n\sqrt{1-\zeta^2}$$



- if $\zeta < 1$, sensor "rings" at ω_d
- these oscillations die out and the expected measurement, KA, is eventually achieved
- system performance can be characterized by *rise time* and *settling time*

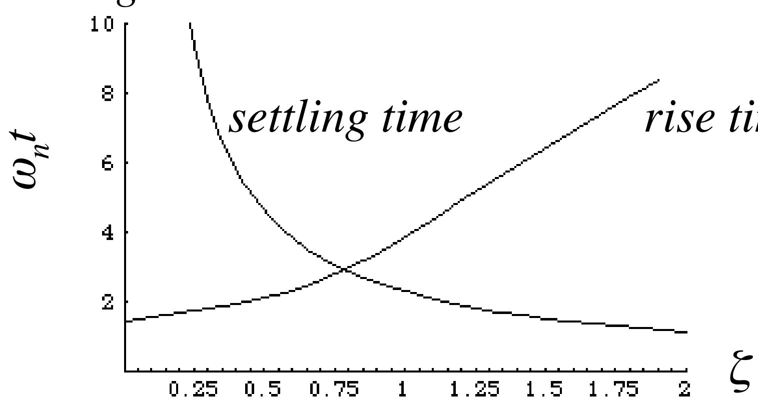
Second-Order Sensors: More Step Input

- As with the first-order sensor, we can compute an error function $\Gamma(t)$:



$$\Gamma(t) = e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right)$$

- *rise time*: time it takes to first achieve 90% of steady-state (or, error is 10%)
- *settling time*: time it takes for oscillations to settle within ±10% of steady-state



- rise time decreases with decreased ζ, but the trade-off is that settling time increases with decreased ζ
- the region near the intersection of these curves is an optimal design point for the sensor

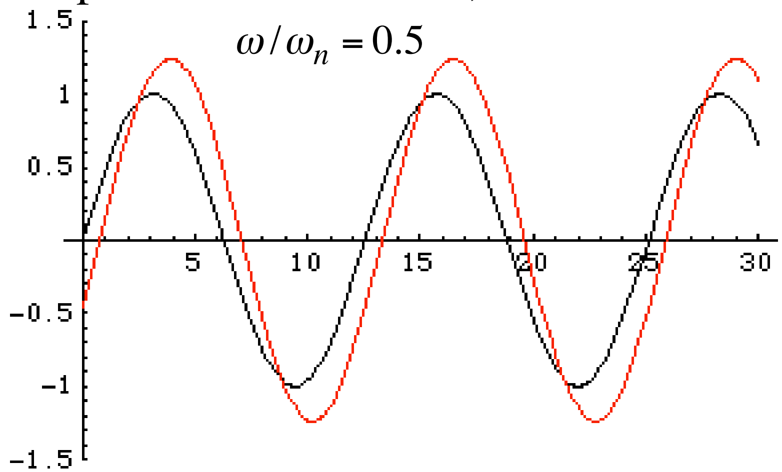
Second-Order Sensors: Periodic Input

- Consider an input that is periodic in time, so $F(t)=A\sin\omega t$

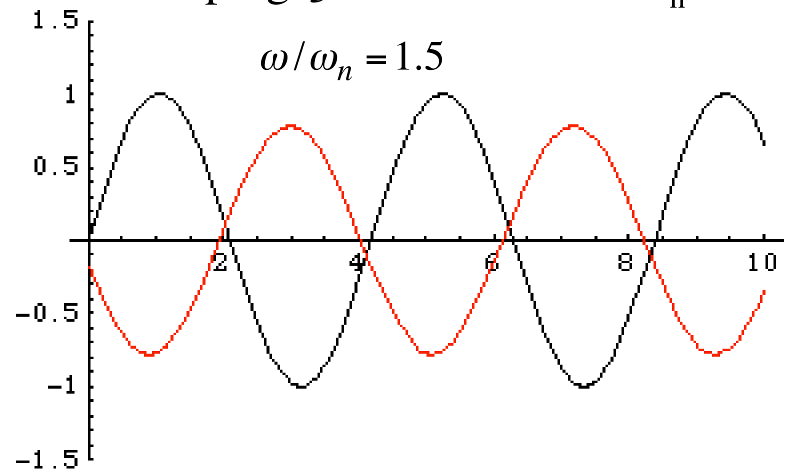
$$y(t) = y_{trans} + \int_{T=0}^{t} A\sin\omega T\,(K\omega_n/\sqrt{1-\xi^2})e^{-\xi\omega_n(t-T)}\sin\omega_d(t-T)dT$$

$$= y_{trans} + \frac{KA\sin(\omega t+\phi)}{\sqrt{\left(1-(\omega/\omega_n)^2\right)^2+(2\xi\omega/\omega_n)^2}}, \text{ where } \tan\phi = \frac{-2\xi\omega/\omega_n}{1-(\omega/\omega_n)^2}$$

- After the transient portion (which oscillates at sensor time scale ω_n) dies out, the sensor response oscillates at the same frequency as the input (ω), but with a phase shift and an amplitude modification, both of which depend on damping ξ and the ratio ω/ω_n



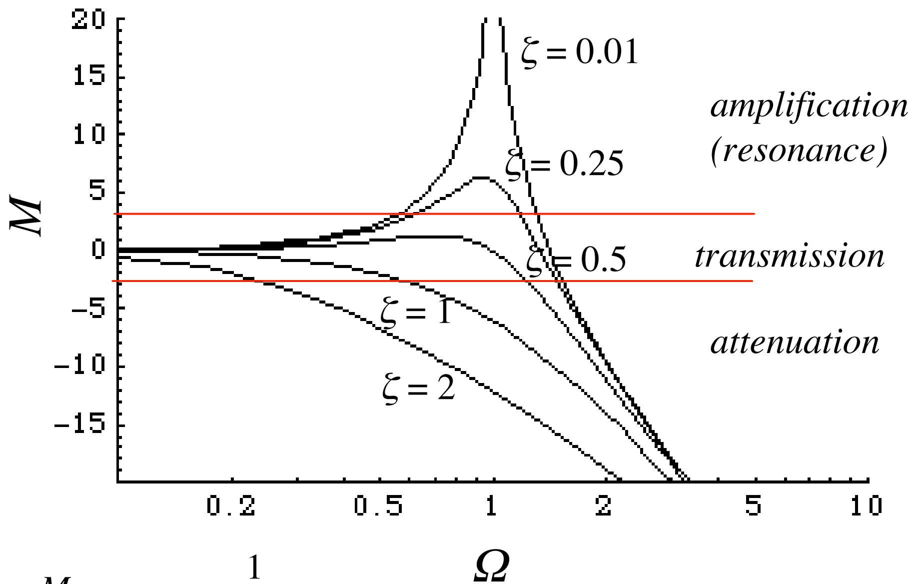
output lags the input and is magnified for $\omega/\omega_n<1$



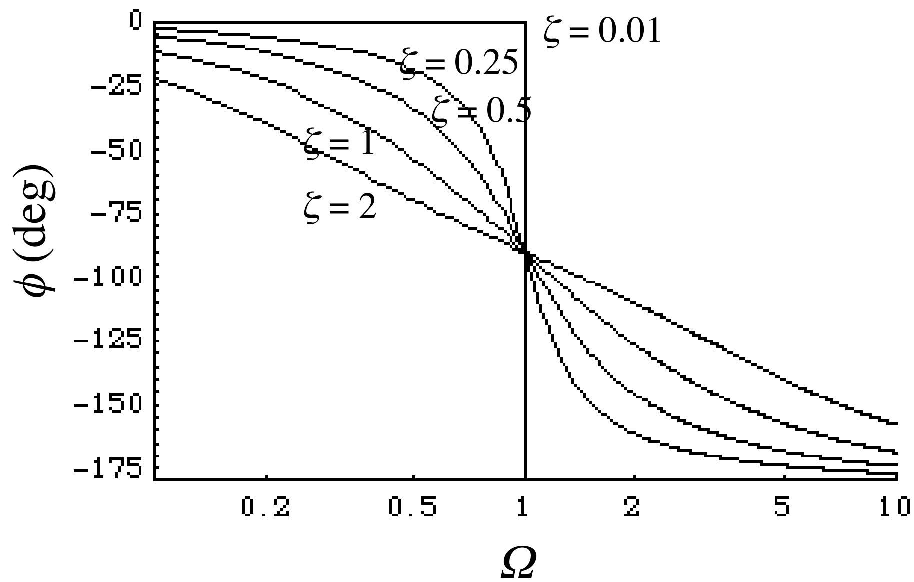
output is attenuated for $\omega/\omega_n>1$, and the lag continues to increase

Second-Order Sensors: Periodic Input (Cont'd)

- This amplitude effect M and phase shift ϕ are shown here; define $\Omega=\omega/\omega_n$



$$M = \frac{1}{\sqrt{\left(1-\Omega^2\right)^2 + (2\zeta\Omega)^2}}$$

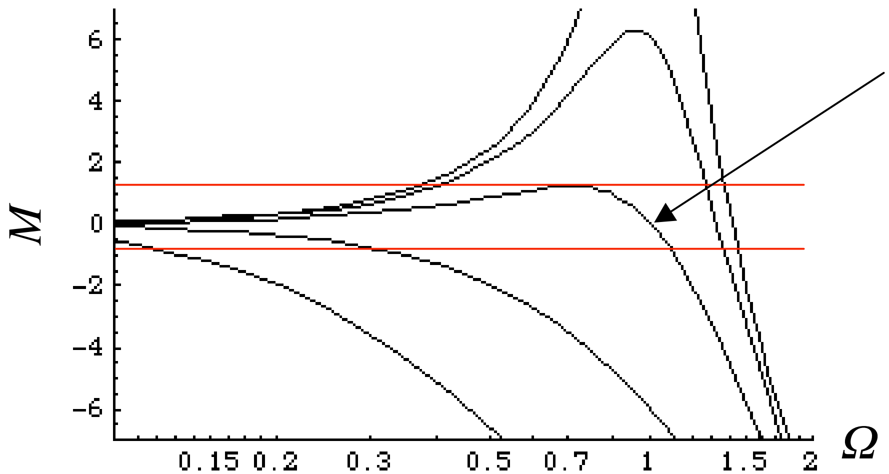


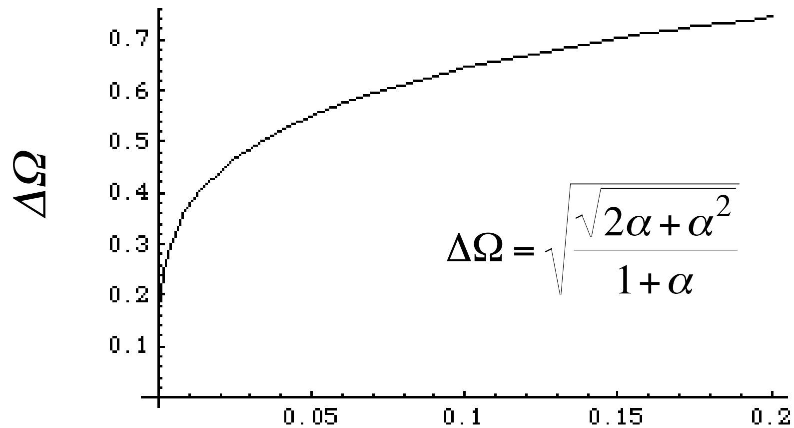
- resonance occurs if input frequency approaches the sensor's natural frequency (large attenuation, especially for small ζ)
- M is near unity for small Ω and goes to zero for large Ω (regardless of ζ)

- phase shift increasingly lags as Ω increases, with the most abrupt changes occurring at resonance (especially for small ζ)
- output is in phase with input for small Ω and out of phase for large Ω (regardless of ζ)

Transmission Band: Unity Gain and Linear Phase Regime

• As shown previously, there is a frequency regime for second-order sensors where transmission is near unity and phase shifts are near zero.





$$\Delta\Omega = \sqrt{\frac{\sqrt{2\alpha+\alpha^2}}{1+\alpha}}$$

error tolerance α

• $\zeta = 0.7$ corresponds to the widest band for retaining a unity transfer function

• this is also the regime where the phase is most closely *linearly* related to the frequency

• nonlinear phase shifts cause a non-uniform phase lead/lag in the original signal and the measured signal (*signal distortion*)

Signal Distortion

- Suppose have original signal to be measured:

$$x(t) = \sin 2\pi t + \sin 4\pi t$$

- A linear phase shift means that the measured signal may be represented by:

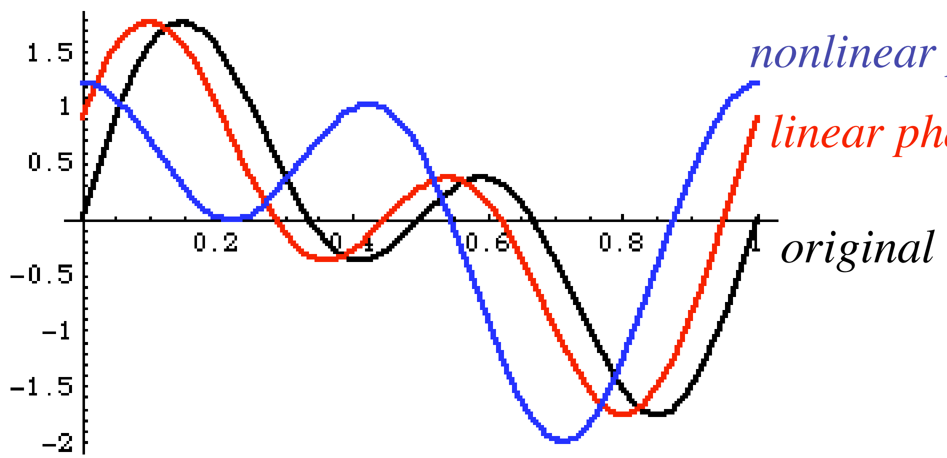
$$y(t) = \sin(2\pi t + \phi) + \sin(4\pi t + 2\phi)$$
$$= \sin\theta + \sin 2\theta$$

which by inspection is the same as the original signal but time-shifted

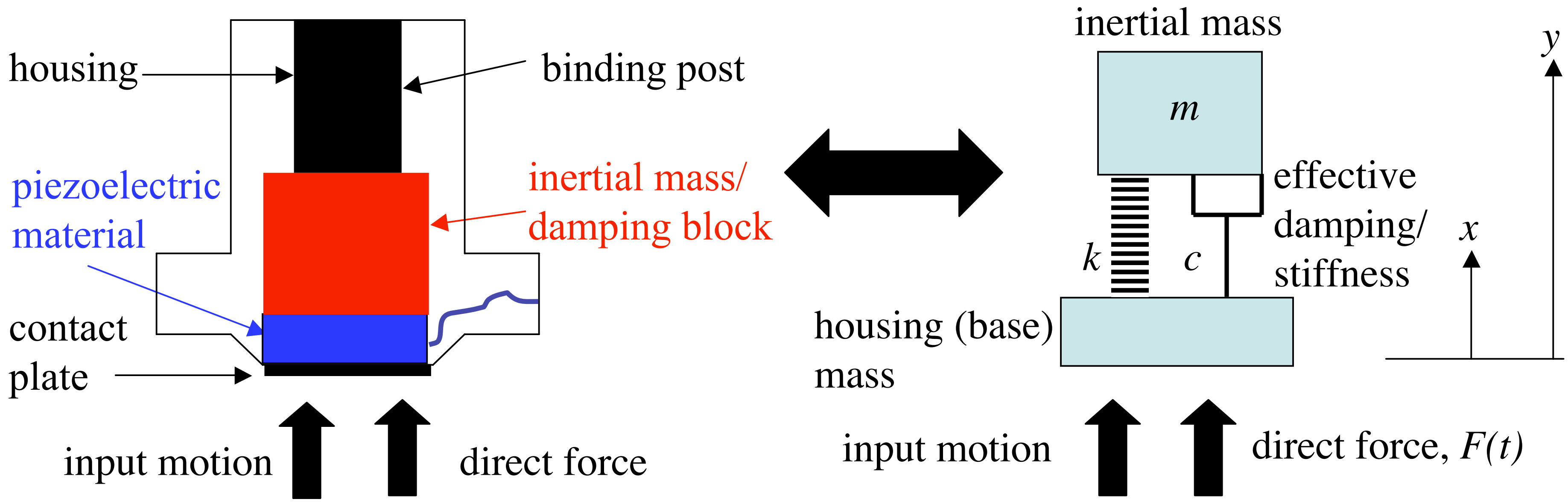
- Consider a nonlinearly phase shifted measured signal:

$$\hat{y}(t) = \sin(2\pi t + 0.1) + \sin(4\pi t + 2)$$

Near sensor resonance, phase distortion is maximized!



Accelerometers: Case of Second-Order Sensor



Newton's 2nd Law on m: $$m\frac{d^2y}{dt^2}+c\frac{dy}{dt}+ky=c\frac{dx}{dt}+kx+F(t)$$

Accelerometers are designed so that the mass is protected from direct forces, so $F(t)=0$; if we define $z=y-x$, we can re-write this equation and normalize as before for second-order systems:

$$\frac{1}{\omega_n^2}\frac{d^2z}{dt^2}+\frac{2\zeta}{\omega_n}\frac{dz}{dt}+\underbrace{z}_{\text{output } z}=-\frac{1}{\omega_n^2}\underbrace{\frac{d^2x}{dt^2}}_{\text{input } x}$$

accelerometers measured acceleration of input by relative motion response of the inertial mass

Let's Consider a Periodic Input to this Accelerometer...

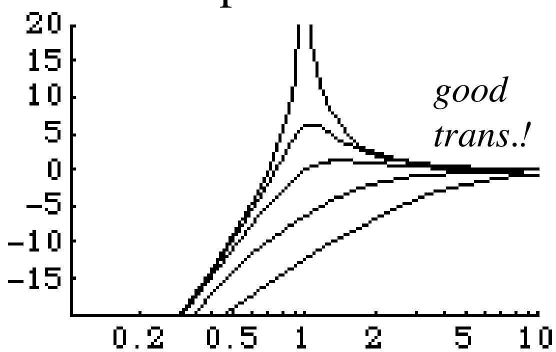
If $x(t)$=$A\sin\omega t$, then $\ddot{x} = -A\omega^2 \sin\omega t$, and

$$z(t) = \frac{KA\Omega^2 \sin(\Omega T - \phi)}{\sqrt{\left(1-\Omega^2\right)^2 + (2\zeta\Omega)^2}}, \quad \tan\phi = -\frac{2\zeta\Omega}{1-\Omega^2}$$

Very response to previous second-order response, but with an extra Ω^2 in the numerator; phase shift ϕ is the same

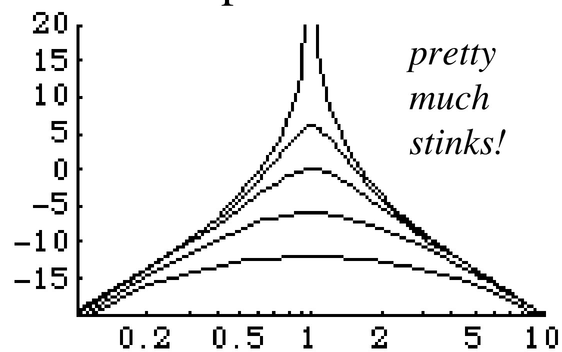
We can define M for this system depending on what we consider the input: displacement, velocity, or acceleration

Displacement input amplitude: A



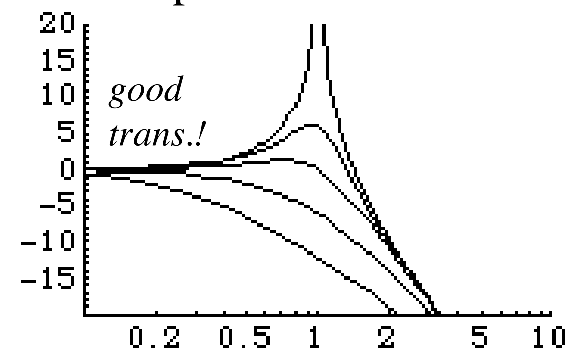
$$M = \frac{\Omega^2}{\sqrt{\left(1-\Omega^2\right)^2 + (2\zeta\Omega)^2}}$$

Velocity input amplitude: $A\Omega$



$$M = \frac{\Omega}{\sqrt{\left(1-\Omega^2\right)^2 + (2\zeta\Omega)^2}}$$

Acceleration input amplitude: $A\Omega^2$

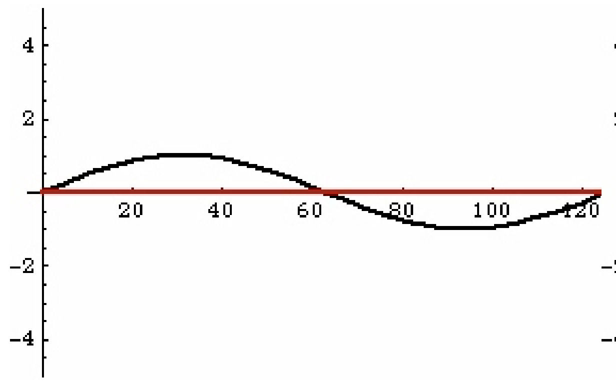


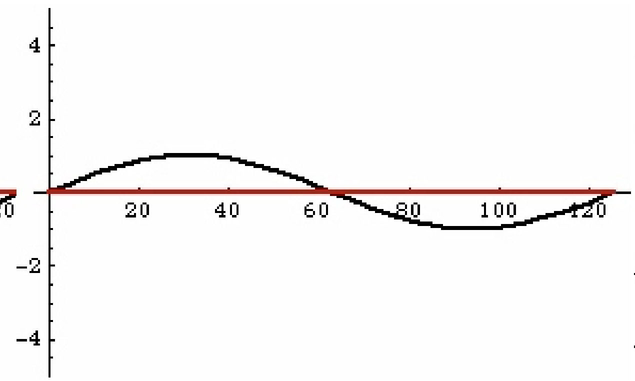
$$M = \frac{1}{\sqrt{\left(1-\Omega^2\right)^2 + (2\zeta\Omega)^2}}$$

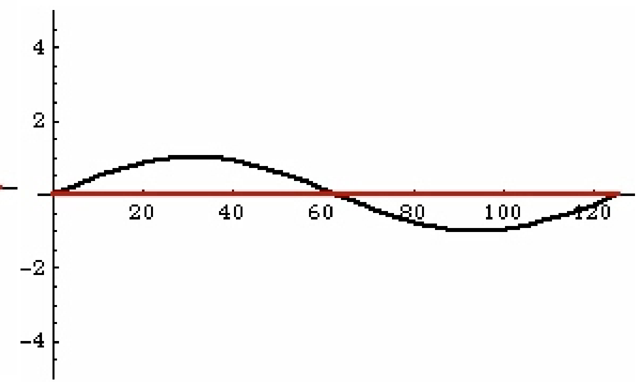
Demonstration of Accelerometer in Displacement Mode

- Consider a periodic displacement input labeled in black) and the corresponding measured output (labeled in red) for three different damping ratios over a swept frequency range ($0<\Omega<8$)

$\zeta = 0.1$ $\zeta = 0.7$ $\zeta = 2.0$







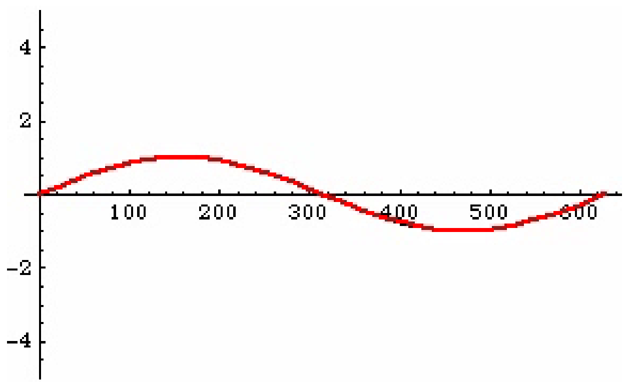
- In displacement mode, output signal is attenuated (but in phase) for low Ω and then enters a near-unity transmission mode for post-resonant Ω (but out-of-phase with input)

- Very low or very high damping shrink the useful transmission band ($\zeta = 0.7$ is optimal)

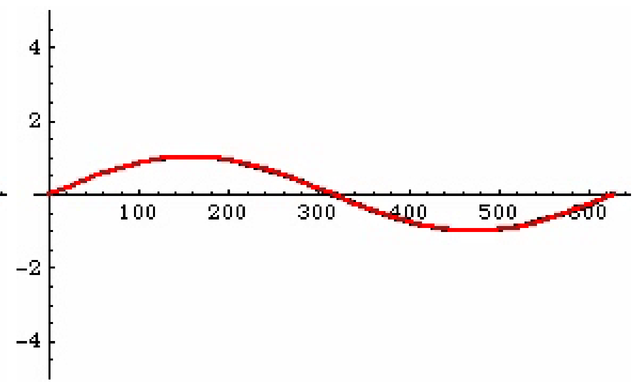
Demonstration of Accelerometer in Acceleration Mode

- Consider a periodic acceleration input labeled in black) and the corresponding measured output (labeled in red) for three different damping ratios over a swept frequency range (0<Ω<8)

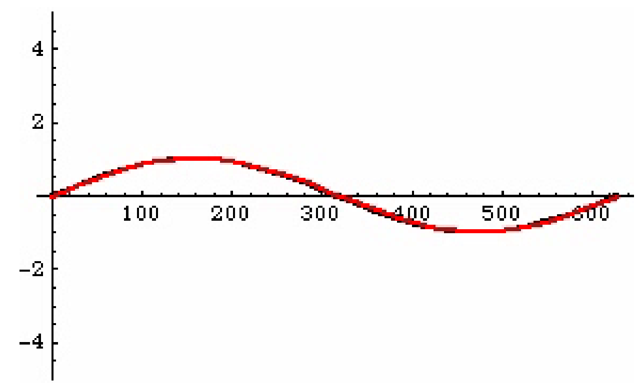
$\zeta = 0.1$



$\zeta = 0.7$



$\zeta = 2.0$



- In acceleration mode, output signal is in phase and in near-untiy transmission for low Ω and then enters attenuated, out-of-phase behavior for post-resonant Ω

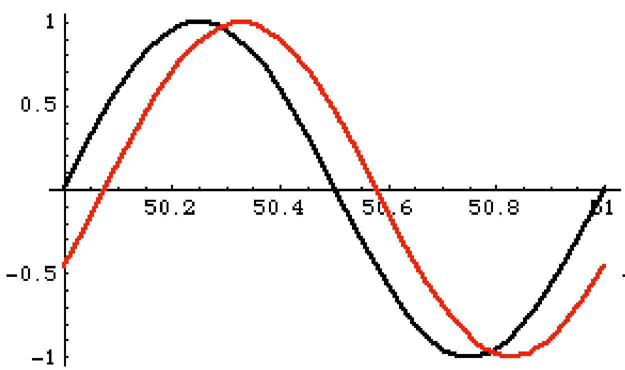
- Very low or very high damping shrink the useful transmission band ($\zeta = 0.7$ is optimal)

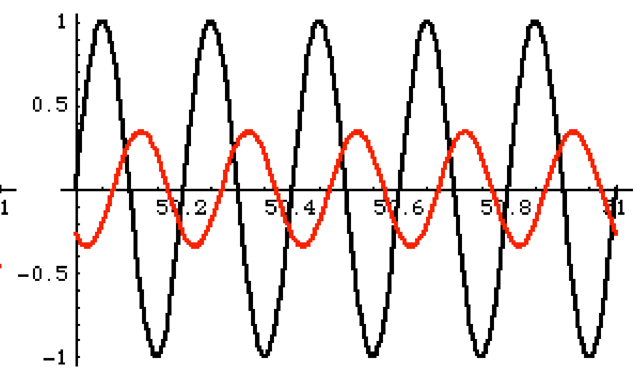
Measuring Wideband Signals Beyond Transmission Band

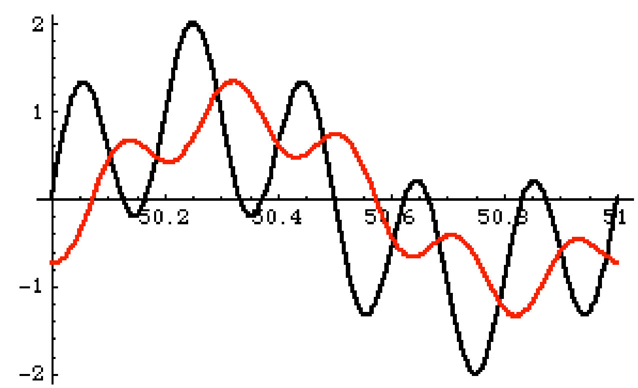
We will assume that the sensor behaves as a linear dynamic system, so multiple inputs may be treated by superposition; suppose a sensor's resonance is at 3 Hz, and we try to measure a signal composed of 1 Hz and 5 Hz components:

$$x(t) = \sin 2\pi t + \sin 10\pi t$$

Input components are in black, outputs in red:







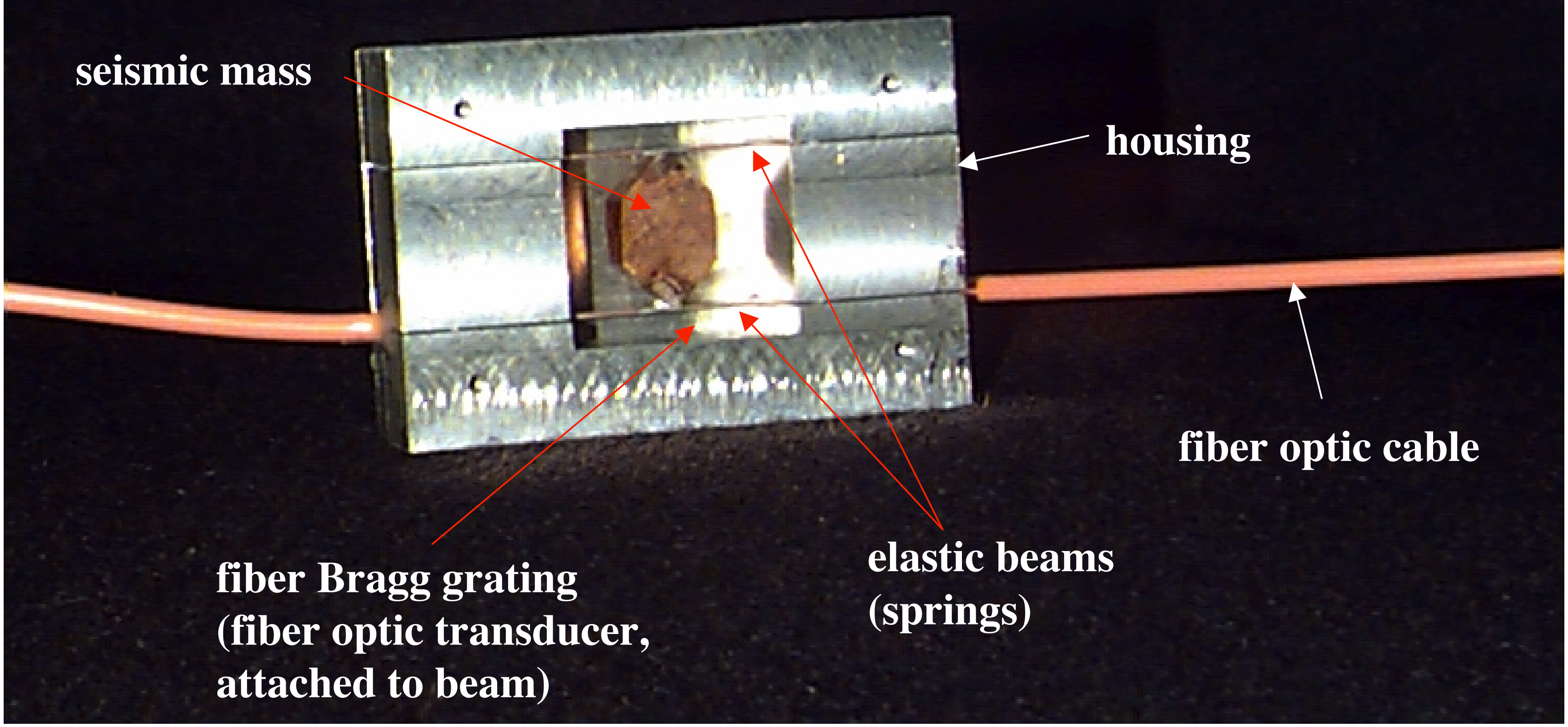
1 Hz component + 5 Hz component = total input and response

Not very good signal representation due to distortions induced by resonance

An Accelerometer Cut-Away

Let's look at a cut-away view of a seismic accelerometer:

- contains seismic mass, as expected
- utilizes elastic beams as springs
- in this example, uses fiber Bragg gratings to transduce bending of beams into displacement/acceleration



Sensor Performance Properties

We will utilize this example of an accelerometer to explore several sensor performance properties:

- *sensitivity*: What is the response of the sensor to inputs, usually over a range of time scales? (Usually given in terms of the transfer function)

- *resolution*: What is the minimum detectable value of the intended input? (Usually given in terms of power or amplitude spectral density)

- *cross-axis sensitivity*: How much does the sensor respond to inputs not aligned with the primary sensing direction? (Usually expressed as a fraction of the main sensitivity)

- *multiple resonances*: Does the sensor have multiple nonlinear (resonant) areas that affect sensitivity and response? (The answer is typically, “yes”).

- *sensitivity to extraneous measurands*: Does the sensor respond to unintended inputs? (Does an accelerometer, for example, also respond to strain or temperature inputs yielding “false” signals?)

Sensitivity

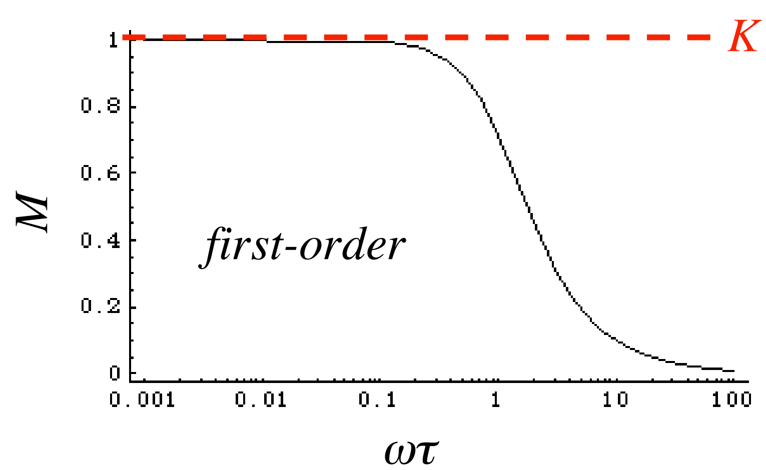
We've already visited sensitivity when we studied the responses of zero, first, and second order instruments to various inputs

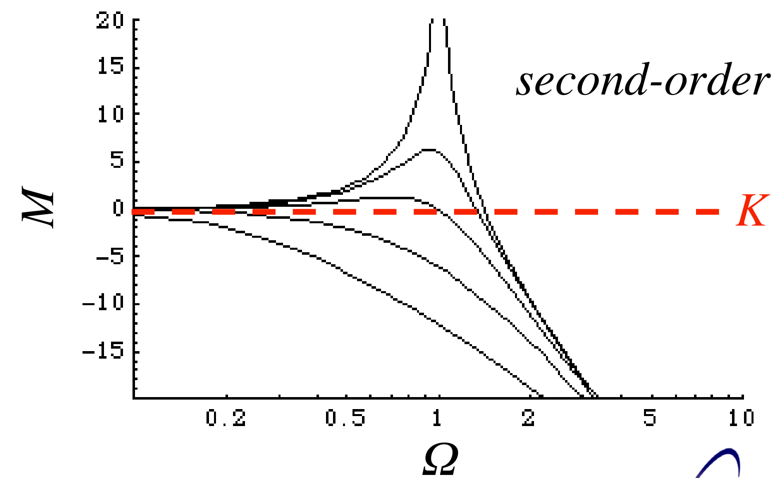
• static sensitivity, K: sensor's static response to a static input; the steady-state response to a unit amount of step input yields a sensor output of K:

$$y(t) = K \qquad y(t) = K + (K - y_0)e^{-t/\tau} \qquad y(t) = K - Ke^{-\zeta\omega_n t}\left(f(\zeta)\sin(\omega_d t + \phi)\right)$$

zero-order *first-order* *second-order*

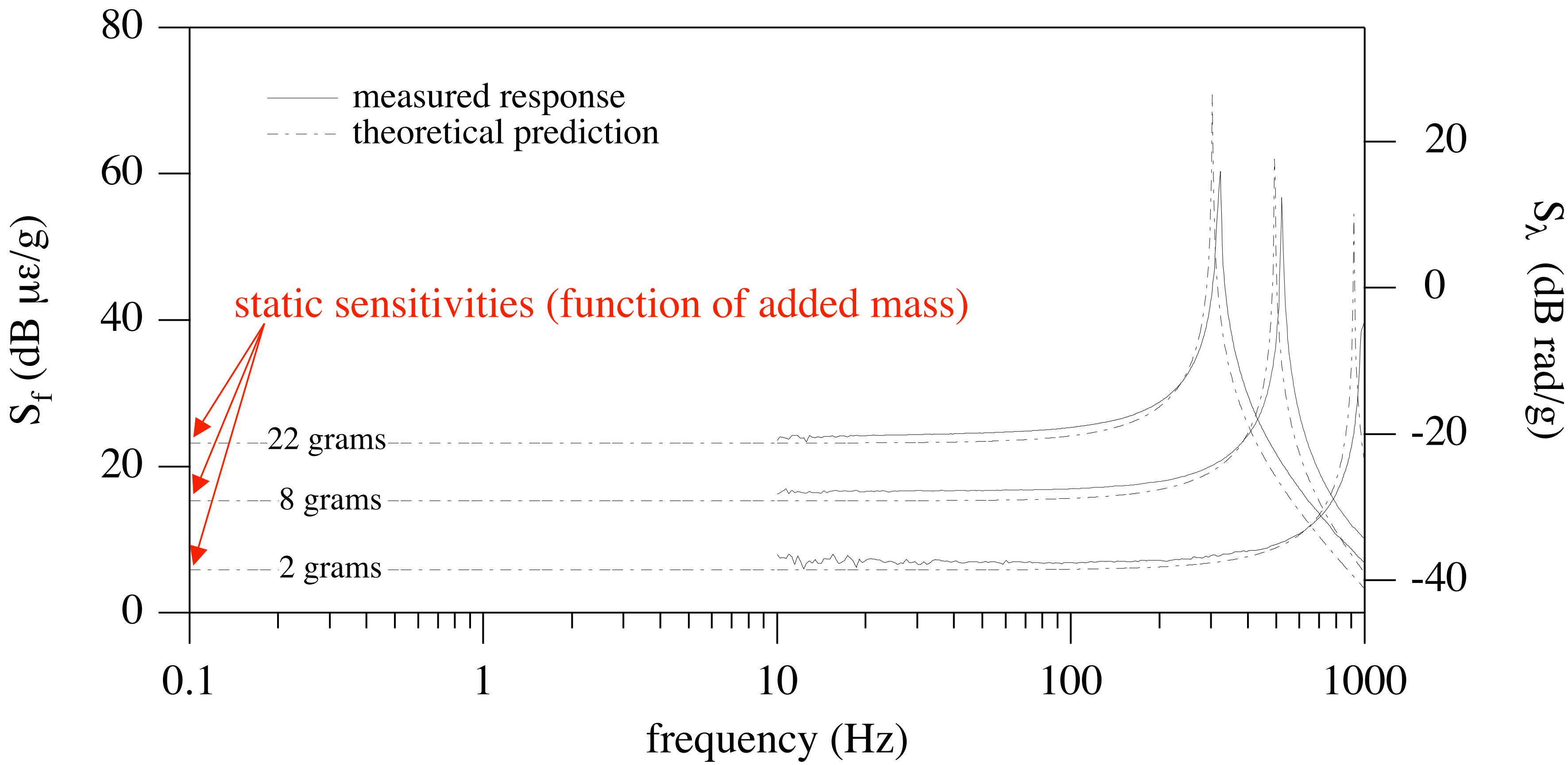
• dynamic sensitivity: the modification of the static sensitivity by dynamical properties of the sensor system; the steady-state response to a unit input at a certain frequency





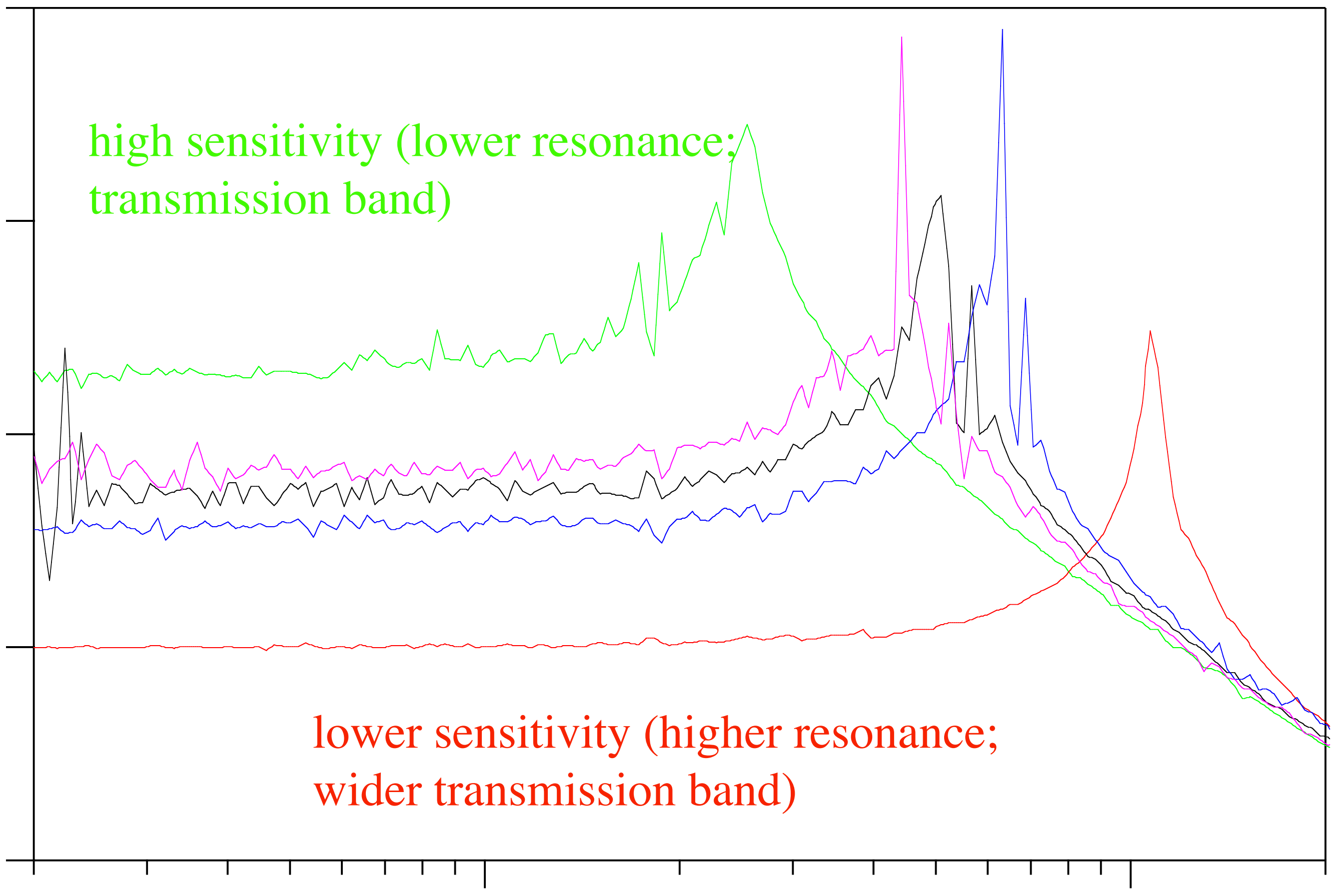
Static and Dynamic Sensitivity of FBG Accelerometer

$$|y(f)| = \frac{K(M, L_{beams}, g, h_{beams})}{\sqrt{\left(f^2 - f_n^2\right)^2 - 4\zeta^2 f^2 f_n^2}}$$



Design Trade-Off: Sensitivity and Bandwidth

The mechanical designs of seismic mechanical motion sensors often involve design trade-offs between performance properties: higher sensitivity often means a lower transmission band:



Resolution

Resolution refers to the sensor's minimum detection capability for the intended input; this property is usually expressed as a power or amplitude spectral density

- Spectral densities may be estimated from finite Fourier transforms of time records

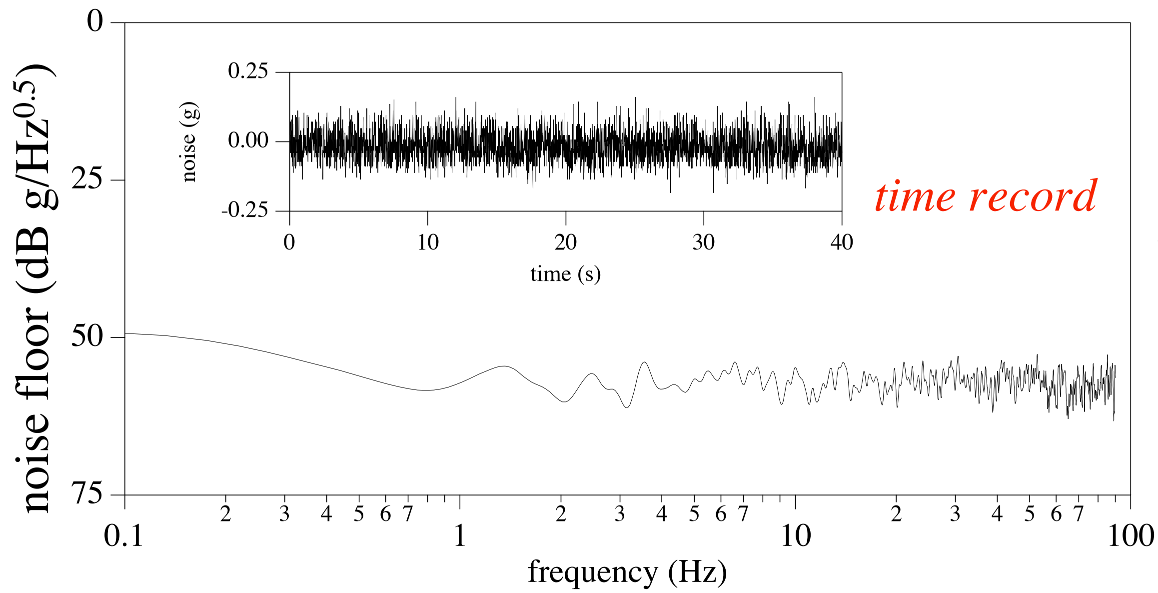
$X(f_k) = \Delta t \sum_{n=0}^{N-1} x_n e^{-2\pi ikn/N}$	Fourier transform for N-point time record (units/Hz)
$G_{XX}(f_k) = \frac{2}{N\Delta t}\left\lvert X(f_k)\right\rvert^2$	power spectral density (units^2 / Hz)
$A_{XX}(f_k) = \sqrt{G_{XX}(f_k)}$	amplitude spectral density (units / Hz$^{1/2}$)

- One way of determining resolution capability is to take data with the sensor in an as quiet environmental state as possible, i.e., no inputs to the sensor

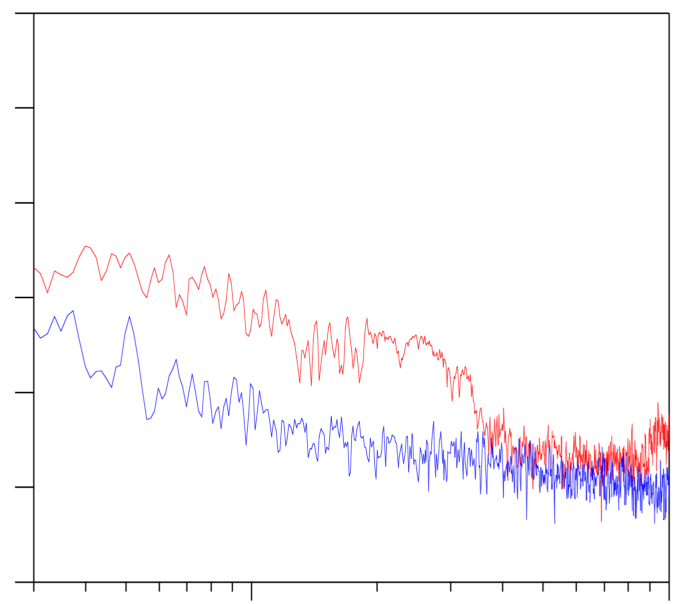
- The practical resolution capability not only includes the sensor dynamics but all aspects of the electronics (amplifier, filter, D/A)

Resolution of FBG Accelerometers

- Accelerometer put in a lead-lined box in the laboratory
- Amplitude spectral density was computed from an ensemble average of several time records



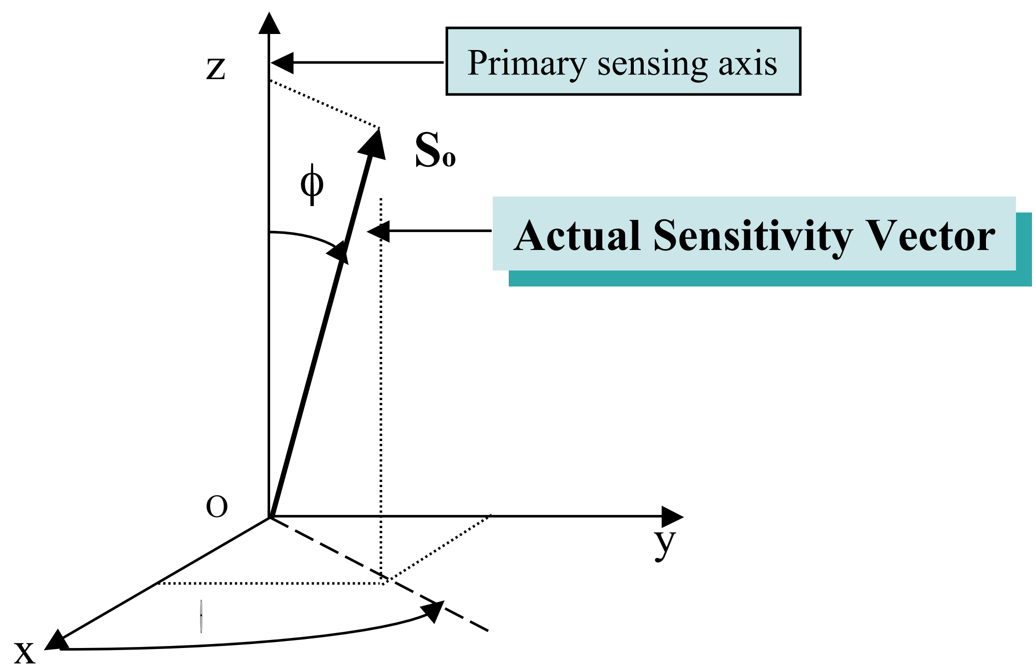
FBG optical accelerometer
- flat response over wide bandwidth
- 3 mg/Hz$^{0.5}$ resolution over all freq.



interferometric optical accelerometer
- 2 μg/Hz$^{0.5}$ resolution at low-mid freq.
- electronic noise rise at low freq.

Cross-Axis Sensitivity

Design imperfections often cause the intended sensing axis to be misaligned with the actual sensitivity vector, leading to cross-axis sensitivity:



The sensitivity vecto rfor a single intended axis may be characterized by its orientation in θ and ϕ:

$$\mathbf{S} = \{K\sin(\phi)\cos(\theta)\}\mathbf{i} + \{K\sin(\phi)\sin(\theta)\}\mathbf{j} + \{K\cos(\phi)\}\mathbf{k}$$

$$\mathbf{S}_{ideal} = K\mathbf{k}$$

Cross-Axis Sensitivity

Given this general sensitivity vector, let's consider a general acceleration input vector, given by $\mathbf{a} = a_x\,\mathbf{i} + a_y\,\mathbf{j} + a_z\,\mathbf{k}$ where we will assume the primary direction is in z.

The measured output M is just the dot-product of these vectors:

$$M = \mathbf{S} \cdot \mathbf{a} = S_{zx}\,a_x + S_{zy}\,a_y + S_{zz}\,a_z \qquad \text{where}$$

$$S_{zx} = K\sin(\phi)\cos(\theta)$$
$$S_{zy} = K\sin(\phi)\sin(\theta)$$

cross-axis terms

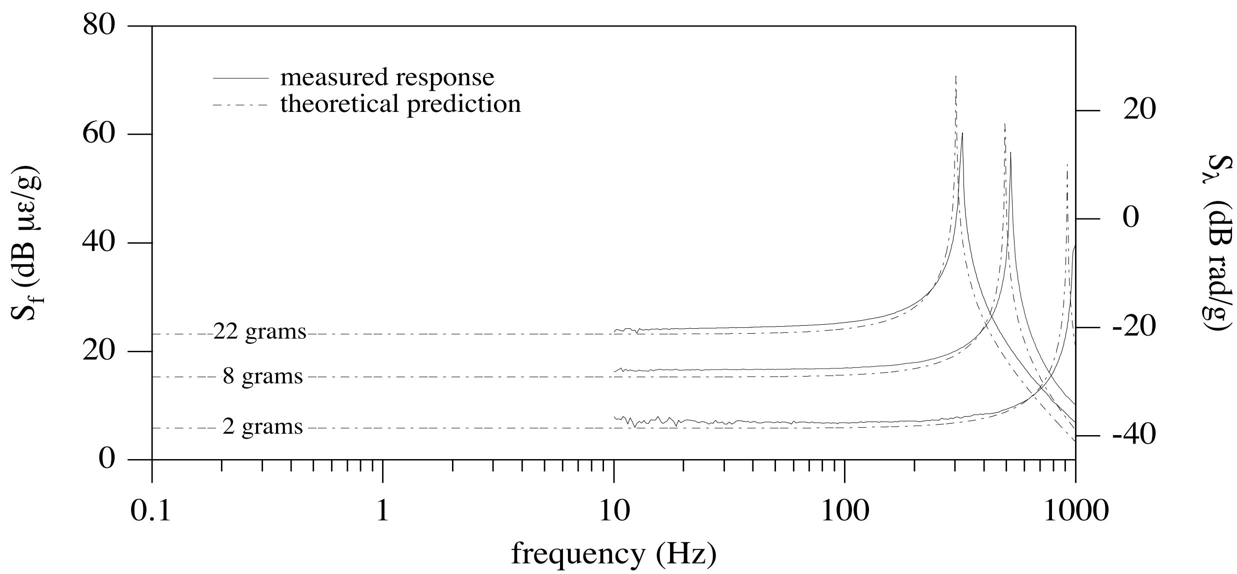
$$S_{zz} = K\cos(\phi)$$

intended axis term

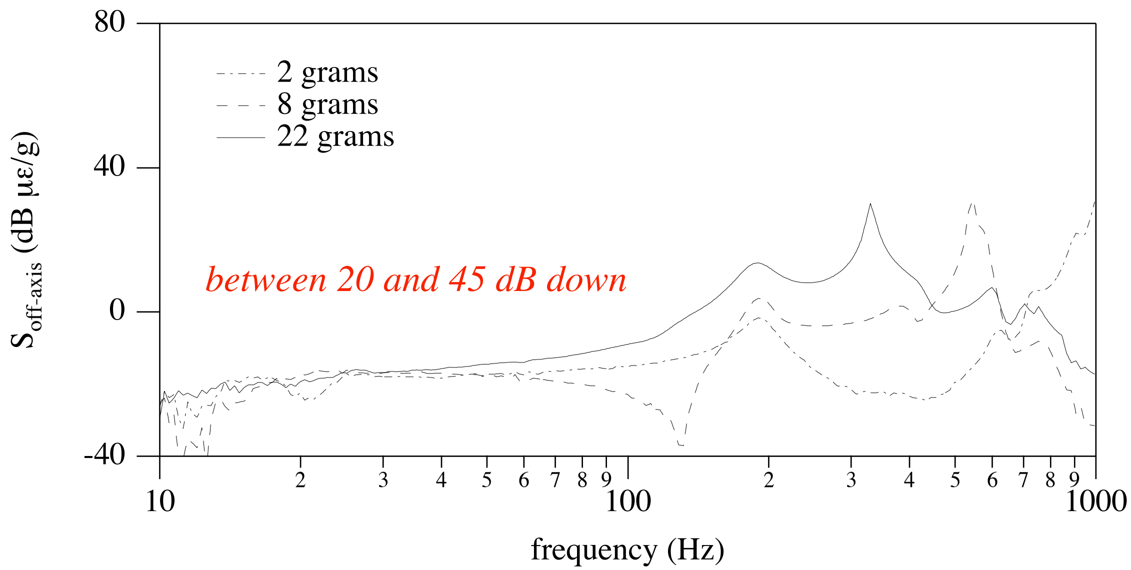
- Ideally, measurement should be $M = Ka_z$
- Accelerations from other directions corrupt measurement
- Even dominant axis sensitivity attenuated (by $\cos\phi$)
- A calibration may be performed to get the sensitivity vector in a least-squares sense; put accelerometer on a shaker (with characterized directionality) on all its sides for N tests:

$$\begin{bmatrix} M_1 \\ \vdots \\ M_N \end{bmatrix} = \begin{bmatrix} a_{x,1} & a_{y,1} & a_{z,1} \\ \vdots & \vdots & \vdots \\ a_{x,N} & a_{y,N} & a_{z,N} \end{bmatrix} \begin{bmatrix} S_{zx} \\ S_{zy} \\ S_{zz} \end{bmatrix} \xrightarrow[\text{least-squares, pseudo-inverse}]{} \text{get } S_{zx}, S_{zy}, S_{zz} \xrightarrow[\text{above equations}]{} \text{get } K, \theta, \phi$$

Cross-Axis Sensitivity



Primary axis sensitivity of FBG accelerometer (from previous viewgraph)

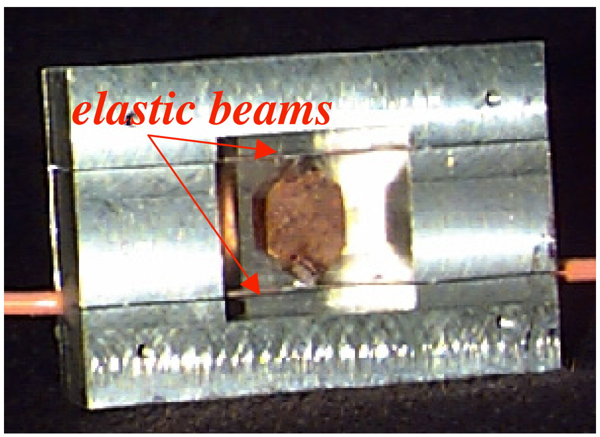


Out-of-plane axis sensitivity of FBG accelerometer (obtained by placing sensor on its side on a shaker)

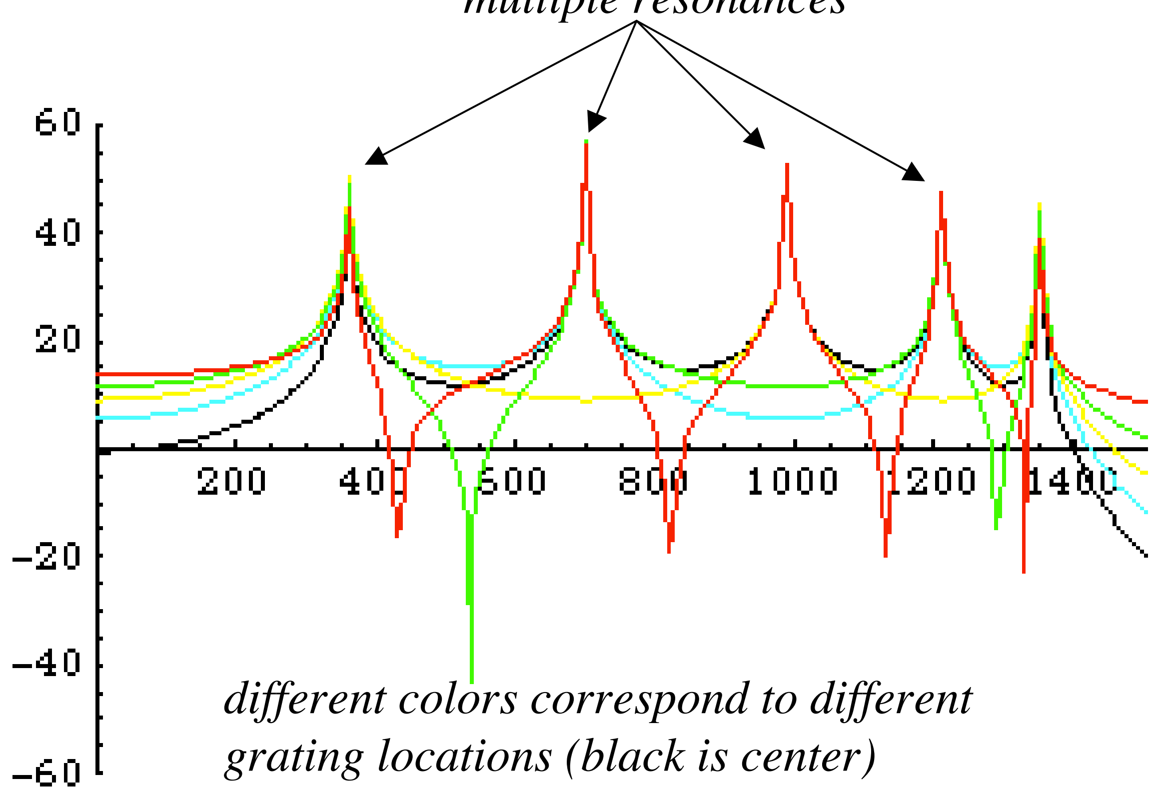
Multiple Resonances

- In reality, most sensor transducer elements comprising the “mass” and “spring” elements are *themselves* continuum structures and thus have multiple resonances

- Case of FBG accelerometer: elastic beams used as springs, and the full-spectrum frequency response diagram is



the beam/mass system has multiple resonances and multiple transmission bands



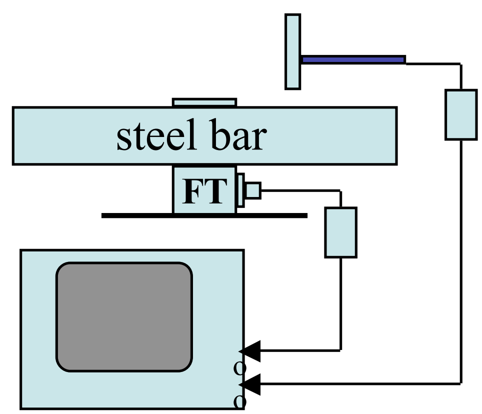
different colors correspond to different grating locations (black is center)

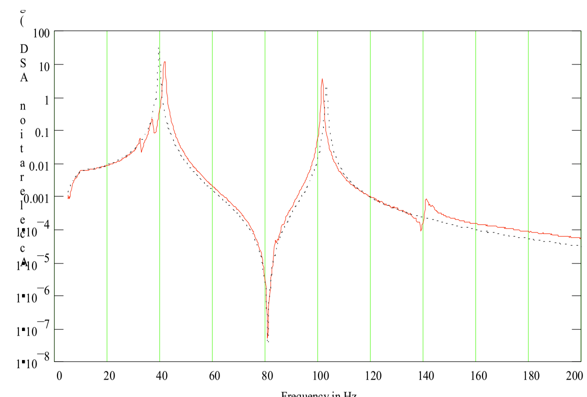
Sensitivity to Extraneous Measurands

- The elements of many sensors are often sensitive to unintended inputs (due to their construction, operation, or connection to other elements), making the sensor produce a signal even when an actual intended signal is not present
- Some examples include temperature, pressure, or strain dependences; for example, with the FBG accelerometer, what if the ambient temperature changed? FBGs are just strain gages, so they will measure temperature.
- Some force transducers (which are just like accelerometers in design!) have bending moment sensitivity and base strain sensitivity...local bends or strains applied at the attachment points can cause a signal because the housing is coupled into the sensor dynamics
- How sensors are attached (and to what they are attached!) are CRITICAL!

*The force transducer is supporting a structure with large inertia, and its modes (translational, bending) are being detected**.

*K. G. McConnell and P. S. Varato, "A Model for Force Transducer Bending Sensitivity and Response During Calibration," Proc. 11th IMAC, pp. 516-521, 1993.





Some Current Trends in Sensing

- SMALLER, SMALLER, SMALLER

✓ Microelectromechanical systems (MEMs) have been and continue to be developed to measure fields such as force, acceleration, and strain
✓ These sensors work on same seismic principles mentioned, just on much smaller scale
✓ The challenges are in engineering parts to required tolerances at these scales (and dealing with significant static electricity build-up problems due to Coulomb interactions)

- Ride the light (with apologies to Qwest, Inc.)

✓ Utilizing fiber optics for various sensing applications began in the 1970s and exploded in the 1990s during the telecom boom
✓ Fiber optic accelerometers and force gages STILL have to utilize seismic design principles
✓ Electronics more complex than conventional sensors...that just means more transfer functions to fully characterize system performance
✓ Huge performance gains in many applications due to eleectromagnetic insensitivity